STUDY OF NATURAL CONVECTION ALONG A VERTICAL SINUSOIDAL WAVY SURFACE: COMBINED HEAT AND MASS TRANSFER

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الملخص

تمت دراسة انتقال الحرارة و الكتلة بالحمل الحر (أو الطبيعي) على الأسطح المتموجة العمودية حيث تكون القوة الطفوية هي نتيجة فرق درجة الحرارة وفرق التركيز. تم تحوير السطح المتموج إلى صفيحة مسطحة واستخدمت طريقة الفرق المحدد الضمني لحل المعادلات الحاكمة المتحصل عليها. اشتقت المعادلات لسطح عمودي ذي شكل اختياري وتم تخصيص النتائج لسطح عمودي ذي تموج جيبي. كما تمت دراسة تأثير كل من رقم براندتل و رقم شميدت والمتسع الموجي للسطح المتموج على معدل انتقال الحرارة والكتلة. زيادة رقم براندتل وجد أنه يزيد من معدل إنتقال الحرارة أما زيادة رقم شميدت فيزيد من معدل انتقال الكتلة ولكن تأثيره على معدل انتقال الحرارة سلبي حيث أن زيادته تخفض من معدل انتقال الحرارة. الزيادة في سعة تموج السطح وجد أنها تنخفض من معدل انتقال الحرارة والكتلة. كما وجد أن متوسط رقم نوسلت ومتوسط رقم شيرود لسطح ذي تموج جيبي دائما يكونان أصغر مما يناظرهما لسطح مسطح.

ABSTRACT

The problem of natural convection along a vertical wavy surface has been studied numerically when mass transfer is combined with heat transfer and the buoyancy force is due to temperature and concentration differences. The wavy surface is transformed to a flat surface for which the obtained governing equations are solved using an implicit finite difference scheme.

The derivations are carried out for a vertical surface of arbitrary shape, and the numerical results are provided for a sinusoidal wavy surface. The sinusoidal wavy surface is used as example because it can be considered as an approximation to practical geometry for which natural convection heat and mass transfer are of interest.

The effects of Prandtl number, Schmidt number, mass transfer number, and the amplitude of the sinusoidal wavy surface on the heat transfer and mass transfer rates are investigated.

Increasing the Prandtl number is found to increase the heat transfer rate. For a given Prandtl number, Increasing Schmidt number decreases the heat transfer rate and increases the mass transfer rate.

Increasing the amplitude of the wavy surface tends to decrease the heat and mass transfer rates and to increase the magnitudes of the local Nusselt number and local Sherwood number. The average Nusselt number and average Sherwood number for sinusoidal wavy surface are found to be smaller than those for the corresponding flat plate.

KEYWARDS: Combined (coupled) heat and mass transfer: Natural (free) convection; vertical plate; Wavy surface.

INTRODUCTION

Many natural convection heat transfer problems of practical interest involve bodies of arbitrary and complex form and configuration. In most cases, the surface generating the flow is curved with an angle between the tangent to the surface and the direction of gravity, varying as the flow proceeds down stream along the surface from its leading edge. In some circumstances, the curvature of the surface is small and it may be approximated as a flat surface, with possibility of employing the information available for vertical and inclined flat surfaces. There is abundant literature for different heating conditions for various kinds of geometries and for different fluids. However, very few studies which consider the effect of complex geometries on natural convection have been reported.

The phenomena of laminar film condensation and laminar film evaporation which occur in numerous industrial processes is an example of practical application of natural convection heat and mass transfer on a sinusoidal wavy surface. The interface is always wavy and the momentum, heat and mass transfer across it is not similar to that across a smooth flat surface. Many studies have been reported concerning the prediction of heat transfer rates accompanying these phenomena, but very few works have considered the effect of the usually present interfacial waves.

Natural convection process involving the combined heat and mass transfer are encountered in many natural processes such as evaporation, condensation, solidification of binary alloy, crystal growth and agricultural drying, and in many industrial applications. The basic problem is governed by the joint action of the buoyancy effects arising from the simultaneous diffusion of thermal energy and of chemical species. Therefore, the continuity, momentum, energy and species equations are coupled through the buoyancy terms, if other effects are neglected.

The heat transfer by natural convection over a vertical wavy surface has been investigated by many authors [1-14]. Yao [1], studied the case of uniform surface temperature laminar free convection along semi-infinite vertical wavy surface, and the effect of surface waviness on the boundary layer was investigated. He found that the local heat transfer rate depends on the slope of the wavy surface and the heat transfer rate for a sinusoidal surface is smaller than that of the corresponding flat surface. The wavy surface effect is found to be small when the amplitude of the wavy surface is completely covered in the boundary layer. Chiu and Chou [4] studied the transient and steady natural convection along a vertical wavy surface in micropolar fluids. Rees and Pop studied the free convection flow along a vertical wavy surface with constant wall temperature [5] and with uniform wall flux [6]. Alam et al (8) have also studied the problem in presence of a transverse magnetic field. The effect of internal heat generation on a steady two dimensional natural convection flow of viscous incompressible fluid along a uniformly heated vertical wavy surface has been investigated by Molla et al[13].

The problem of combined effects of thermal and mass diffusion in natural convection flow along vertical flat plate has been studied rather extensively [15-23]. In contrast to the vertical flat surface, the problem of natural convection on vertical wavy surface with combined effect of heat and mass transfer seems not has been well investigated. This motivated the present work.

The combined heat and mass transfer by natural convection over a vertical wavy surface has been investigated by Hossain and Rees [24]. They studied the effects of different values of the governing parameters such as Schmidt number, amplitude of the waviness of the surface on the rate of heat and mass transfer. The transient behaviors of natural convection heat and mass transfer along a vertical wavy surface subjected to step changes of wall temp and wall concentration have been investigated by Jang and Yan [25]. In their work, the effects of wavy geometry, buoyancy ratio, and Schmidt number of the transient local skin friction, local Nusselt number and local Sherwood number have been discussed. Cheng [26] studied the phenomenon of natural convection heat and mass transfer near a vertical wavy surface embedded in a fluid-saturated porous medium. He has also investigated coupled heat and mass transfer by natural convection flow along a wavy conical surface and vertical wavy surface in a porous medium [27].

Most of the previous studies about vertical wavy surfaces are concerned with micro-fluids or porous media. Natural convection heat and mass transfer in Newtonian fluid along a vertical wavy surface has not been well investigated.

In this study, the natural convection heat and mass transfer along a vertical wavy surface is examined numerically. The effect of Prandtl number, Schmidt number and the dimensionless amplitude of the wavy surface on the local Nusselt number and local Sherwood number are studied.

The transformation method proposed by Yao [1], in his study of natural convection heat transfer from isothermal vertical wavy (sinusoidal) surface in newtonian fluids, is adopted in this work.

PROBLEM FORMULATION

In this work, the transformation method proposed by Yao [1], is used to transform a sinusoidal wavy surface into a flat surface. The problem involves a natural convection on a semi-infinite isothermal vertical wavy plate with a temperature and mass fraction different from that of the ambient as shown in Figure (1).

The thermo-physical properties are assumed to be constant except for density variations in the buoyancy term in the x-momentum equation. The Boussinesq approximation is used to characterize the buoyancy effect. The surface of the plate is described by;

$$y = \overline{x}(\overline{\sigma}) \tag{1}$$

The plate is situated in an otherwise quiescent fluid with a temperature T_∞ and mass fraction C_∞. The plate is maintained at constant temperature T_w, and constant mass fraction C_w. The characteristic length associated with the wavy surface is ℓ. The x-coordinate is measured from the leading edge of the plate and the y-coordinate is measured normal to the x-coordinate.

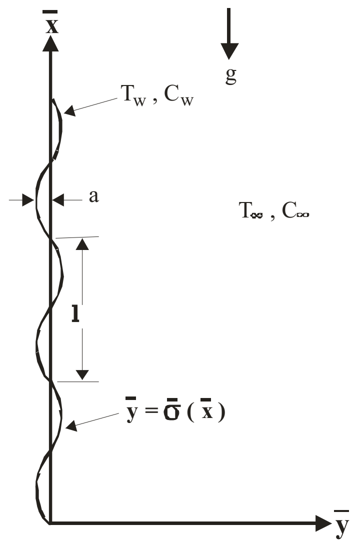

Figure 1: Schematic diagram of the physical system

The dimensional governing equations are the continuity, the Navier- Stockes equations, the energy equation and the species equation in two-dimensional Cartesian coordinates(x,y). The flow is assumed to be steady, and the fluid to Newtonian with constant properties except for density in the momentum equation (Boussinesq approximation). Five differential equations in dimensional form describe the dynamics of the system:

Continuity equation,

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0 \tag{2a}$$

x-momentum equation,

$$\rho \bar{u} \frac{\partial \bar{u}}{\partial \bar{x}} + \rho \bar{v} \frac{\partial \bar{u}}{\partial \bar{y}} = -\frac{\partial P}{\partial \bar{x}} + \mu\left(\frac{\partial^2 \bar{u}}{\partial \bar{x}^2} + \frac{\partial^2 \bar{u}}{\partial \bar{y}^2}\right) - \rho g \tag{2b}$$

y-momentum equation,

$$\rho \bar{u} \frac{\partial \bar{v}}{\partial \bar{x}} + \rho \bar{v} \frac{\partial \bar{v}}{\partial \bar{y}} = -\frac{\partial P}{\partial \bar{y}} + \mu\left(\frac{\partial^2 \bar{v}}{\partial \bar{x}^2} + \frac{\partial^2 \bar{v}}{\partial \bar{y}^2}\right) \tag{2c}$$

energy equation,

$$\bar{u} \frac{\partial \bar{T}}{\partial \bar{x}} + \bar{v} \frac{\partial \bar{T}}{\partial \bar{y}} = \alpha\left(\frac{\partial^2 \bar{T}}{\partial \bar{x}^2} + \frac{\partial^2 \bar{T}}{\partial \bar{y}^2}\right) \tag{2d}$$

species equation,

$$\bar{u} \frac{\partial \bar{C}}{\partial \bar{x}} + \bar{v} \frac{\partial \bar{C}}{\partial \bar{y}} = D\left(\frac{\partial^2 \bar{C}}{\partial \bar{x}^2} + \frac{\partial^2 \bar{C}}{\partial \bar{y}^2}\right) \tag{2e}$$

For $\beta \Delta T \langle\langle 1$ and $\beta^* \Delta C \langle\langle 1$, where ΔT and ΔC are temperature and concentration differences, the Boussinesq approximation may be employed, giving;

$$g(\rho_\infty - \rho) = g\beta\rho(T - T_\infty) = g\beta^*\rho(C - C_\infty) \tag{3}$$

Where, $\beta = -\frac{1}{\rho}(\frac{\partial \rho}{\partial T})_{P,C}$ and $\beta^* = -\frac{1}{\rho}(\frac{\partial \rho}{\partial C})_{T,P}$

In the momentum equation, the local static pressure P may be broken down into two terms; one due to the hydrostatic pressure in the ambient medium, P_∞, and the other due to motion of the fluid, P,

$$P = P_\infty + \overline{P} \tag{4}$$

where $P_\infty = -\rho_\infty g\overline{x}$ if the gravitational force is acting in the negative x-direction.

Hence, $$\frac{\partial P}{\partial \overline{x}} = \rho_\infty g + \frac{\partial \overline{P}}{\partial \overline{x}} \tag{5}$$

And $$\frac{\partial P}{\partial \overline{y}} = \frac{\partial \overline{P}}{\partial \overline{y}} \tag{6}$$

Using (3), (5) and (6), the x-momentum equation becomes,

$$\overline{u}\frac{\partial \overline{u}}{\partial \overline{x}} + \overline{v}\frac{\partial \overline{u}}{\partial \overline{y}} = -\frac{1}{\rho}\frac{\partial P}{\partial \overline{x}} + \nu(\frac{\partial^2 \overline{u}}{\partial \overline{x}^2} + \frac{\partial^2 \overline{u}}{\partial \overline{y}^2}) + g\beta(T - T_\infty) + g\beta^*(C - C_\infty) \tag{7}$$

and the y-momentum equation becomes,

$$\overline{u}\frac{\partial \overline{v}}{\partial \overline{x}} + \overline{v}\frac{\partial \overline{v}}{\partial \overline{y}} = -\frac{1}{\rho}\frac{\partial P}{\partial \overline{y}} + \nu(\frac{\partial^2 \overline{v}}{\partial \overline{x}^2} + \frac{\partial^2 \overline{v}}{\partial \overline{y}^2}) \tag{8}$$

It is clear that the buoyancy term in the x-momentum equation is due to the combined effect of thermal and mass diffusion.

Equations (2a), (7), (8), (2d) and (2e), can be reduced to five dimensionless equations using the following dimensionless groups,

$$x = \frac{\overline{x}}{\ell}, \quad r = \frac{\overline{y} - \overline{\sigma}}{\ell}Gr^{\frac{1}{4}} \tag{9a}$$

$$P = \frac{\overline{P}\ell^2}{\rho\nu^2 Gr} \tag{9b}$$

$$\hat{u} = \frac{\overline{u}\ell}{\nu Gr^{\frac{1}{2}}}, \quad \hat{v} = (\frac{\overline{v} - \sigma'\overline{u}}{\nu Gr^{\frac{1}{4}}})\ell \tag{9c}$$

$$\theta = \frac{T - T_\infty}{T_w - T_\infty}, \quad \psi = \frac{C - C_\infty}{C_w - C_\infty} \tag{9d}$$

$$\sigma' = \frac{d\overline{\sigma}}{d\overline{x}} = \frac{d\sigma}{dx} \tag{9e}$$

$$Gr = \frac{g\ell^3\beta(T_w - T_\infty)}{\nu^2}, \quad Gr_c = \frac{g\ell^3\beta^*(C_w - C_\infty)}{\nu^2} \tag{9f}$$

After ignoring terms of small order in Gr, the following equations are obtained:

$$\frac{\partial \hat{u}}{\partial x} + \frac{\partial \hat{v}}{\partial r} = 0 \tag{10a}$$

$$\hat{u}\frac{\partial \hat{u}}{\partial x} + \hat{v}\frac{\partial \hat{v}}{\partial r} = -\frac{\partial P}{\partial x} + Gr^{\frac{1}{4}}\sigma'\frac{\partial P}{\partial r} + (\theta + R\psi) + (1 + \sigma'^2)\frac{\partial^2 \hat{u}}{\partial r^2} \tag{10b}$$

$$\sigma''\hat{u}^2 + \sigma'(\theta + R\psi) = \sigma'\frac{\partial P}{\partial x} - Gr^{\frac{1}{4}}(1+\sigma'^2)\frac{\partial P}{\partial r} \tag{10c}$$

$$\hat{u}\frac{\partial \theta}{\partial x} + \hat{v}\frac{\partial \theta}{\partial r} = \frac{1+\sigma'^2}{Pr}\frac{\partial^2 \theta}{\partial r^2} \tag{10d}$$

$$\hat{u}\frac{\partial \psi}{\partial x} + \hat{v}\frac{\partial \psi}{\partial r} = \frac{1+\sigma'^2}{Sc}\frac{\partial^2 \psi}{\partial r^2} \tag{10e}$$

where, $\quad R = \dfrac{Gr_c}{Gr}$

Equation (10a), is the key step for the transformation that proposed by Yao [1] to transform the wavy surface into a flat surface in which a rectangular computational grid can be fitted in the transformed coordinates (x , r).

Elimination of $\dfrac{\partial P}{\partial r}$ between (11b) and (11c) results in four equations which can be solved for u, v, θ and ψ.

The four governing equations in dimensionless form are:

$$(4x)\frac{\partial u}{\partial x} + (2u - y)\frac{\partial u}{\partial y} + \frac{\partial v}{\partial y} = 0 \tag{11a}$$

$$(4x)u\frac{\partial u}{\partial x} + (v - yu)\frac{\partial u}{\partial y} + (2 + \frac{4x\sigma'\sigma''}{1+\sigma'^2})u^2 = \frac{(\theta + R\psi)}{1+\sigma'^2} + (1+\sigma'^2)\frac{\partial^2 u}{\partial y^2} \tag{11b}$$

$$(4x)u\frac{\partial \theta}{\partial x} + (v - yu)\frac{\partial \theta}{\partial y} = \frac{1+\sigma'^2}{Pr}\frac{\partial^2 \theta}{\partial y^2} \tag{11c}$$

$$(4x)u\frac{\partial \psi}{\partial x} + (v - yu)\frac{\partial \psi}{\partial y} = \frac{1+\sigma'^2}{Sc}\frac{\partial^2 \psi}{\partial y^2} \tag{11d}$$

where, $$x = \frac{\bar{x}}{\ell}, \qquad y = \frac{r}{(4x)^{\frac{1}{4}}} \tag{12a}$$

$$u = \frac{\hat{u}}{(4x)^{\frac{1}{2}}}, \qquad v = (4x)^{\frac{1}{4}}\hat{v} \tag{12b}$$

The velocity components, u and v are parallel to x and y axes, respectively, and is neither parallel nor perpendicular to the wavy surface.

The boundary conditions are,

At $y = 0 \qquad u = v = 0 \quad$ and $\quad \theta = \psi = 1$

At $y \to \infty \qquad u \to 0, \quad \theta \to 0 \quad$ and $\quad \psi \to 0$

The boundary condition v = 0 at y = 0, is strictly valid only for low mass transfer rate. For high mass transfer rate, the velocity, v, at the surface can be derived as follow:

The surface of the wavy surface is described by equation (1). The equation describing the position of the surface can be written as,

$$F(\bar{x}, \bar{y}) = \bar{y} - \bar{\sigma}(\bar{x}) \tag{13}$$

From this relation we can obtain Grad (F) and then the unit vectors normal and tangent to the wall;

$$\vec{n} = \frac{\nabla F}{|\nabla F|} = \frac{\sigma' i + j}{(1+\sigma'^2)^{\frac{1}{2}}} \tag{14}$$

$$\vec{t} = \frac{i + \sigma' j}{(1+\sigma'^2)^{\frac{1}{2}}} \tag{15}$$

The velocity vector of the surface in $(\bar{x}, \bar{y})$ coordinates is,

$$\vec{w} = \bar{u} i + \bar{v} j \tag{16}$$

The velocity normal to the wavy surface is,

$$\bar{w}_n = \vec{w}.\vec{n} = \frac{\bar{v} - \sigma' \bar{u}}{(1+\sigma'^2)^{\frac{1}{2}}} \tag{17}$$

But, $$\bar{v} - \sigma' \bar{u} = v\left(\frac{\beta g \Delta T \nu^2}{4\bar{x}}\right)^{\frac{1}{4}} \tag{18}$$

Therefore, $$\bar{w}_n = \left(\frac{\beta g \Delta T \nu^2}{4\bar{x}}\right)^{\frac{1}{4}} \frac{v}{(1+\sigma'^2)^{\frac{1}{2}}} \tag{19}$$

Where, v is the velocity normal to the x-axis.
Now, v can be written in terms of the normal velocity as,

$$v = (1+\sigma'^2)^{\frac{1}{2}} \frac{\bar{w}_n}{\left(\frac{\beta g \Delta T \nu^2}{4\bar{x}}\right)^{\frac{1}{4}}} \tag{20}$$

or $$v = (1+\sigma'^2)^{\frac{1}{2}} w_n \tag{21}$$

Where, $$w_n = \frac{\bar{w}_n}{\left(\frac{\beta g \Delta T \nu^2}{4\bar{x}}\right)^{\frac{1}{4}}} \tag{22}$$

The total mass flux of certain species at the wall is given by,

$$N\big|_w = -\rho D \left.\frac{\partial C}{\partial n}\right|_w + (C \ N)_w \tag{23}$$

where, $\frac{\partial C}{\partial n}$ is the concentration gradient normal to the wavy surface.
Equation (23) can be written as

$$N\big|_w = -\frac{\rho D \left.\frac{\partial C}{\partial n}\right|_w}{1 - C_w} \tag{24}$$

where, $$\frac{\partial C}{\partial n} = \nabla \vec{C}.\vec{n} = \frac{\left(-\sigma' \frac{\partial C}{\partial \bar{x}} + \frac{\partial C}{\partial \bar{y}}\right)}{(1+\sigma'^2)^{\frac{1}{2}}} \tag{25}$$

using the transformation parameters from ($\bar{x}$, $\bar{y}$) to (x , r), we obtain,

$$\frac{\partial C}{\partial \bar{x}} = \frac{1}{\ell}\frac{\partial C}{\partial x} - \frac{\sigma'}{\ell} Gr^{\frac{1}{4}} \frac{\partial C}{\partial r} \tag{26}$$

and $$\frac{\partial C}{\partial \bar{y}} = \frac{Gr^{\frac{1}{4}}}{\ell}\frac{\partial C}{\partial r} \tag{27}$$

the concentration gradient tangent to the wavy surface,

$$\frac{\partial C}{\partial t} = \nabla \vec{C}.\vec{t} = 0 \tag{28}$$

i.e., $$\left(\frac{\partial C}{\partial \bar{x}} i + \frac{\partial C}{\partial \bar{y}} j\right).\frac{i + \sigma' j}{(1+\sigma'^2)^{\frac{1}{2}}} = 0 \tag{29}$$

or $$\frac{\partial C}{\partial \bar{x}} + \sigma' \frac{\partial C}{\partial \bar{y}} = 0 \tag{30}$$

Substituting of (26) and (27) in (30) gives,

$$\frac{\partial C}{\partial x} = 0 \tag{31}$$

using (26), (27) and (31), we obtain,

$$\frac{\partial C}{\partial n} = (1+\sigma'^2)^{\frac{1}{2}} \frac{Gr^{\frac{1}{4}}}{\ell}\frac{\partial C}{\partial r} = (1+\sigma'^2)^{\frac{1}{2}} \frac{Gr^{\frac{1}{4}}}{\ell(4x)^{\frac{1}{4}}}\frac{\partial C}{\partial y} \tag{32}$$

Or $$\left.\frac{\partial C}{\partial n}\right|_w = (1+\sigma'^2)^{\frac{1}{2}} \frac{Gr^{\frac{1}{4}}}{\ell(4x)^{\frac{1}{4}}}(C_w - C_\infty)\left.\frac{\partial \psi}{\partial y}\right|_{y=0} \tag{33}$$

and hence, equation (24) becomes,

$$N|_w = \frac{\rho D}{(1-C_w)}(1+\sigma'^2)^{\frac{1}{2}} \frac{Gr^{\frac{1}{4}}}{\ell(4x)^{\frac{1}{4}}}(C_w - C_\infty)\left.\frac{\partial \psi}{\partial y}\right|_{y=0} \tag{34}$$

since $N|_w = \rho \bar{w}_n|_w$ and $$v|_w = (1+\sigma'^2)^{\frac{1}{2}} \frac{\bar{w}_n|_w}{\left(\frac{\beta g \Delta T \nu^2}{4\bar{x}}\right)^{\frac{1}{4}}} \tag{35}$$

Therefore, $$v|_w = -\frac{B}{Sc}(1+\sigma'^2)\left.\frac{\partial \psi}{\partial y}\right|_{y=0} \tag{36}$$

Where, $$B = \frac{C_w - C_\infty}{1 - C_w} \tag{37}$$

The parameter B is called mass transfer number; a dimensionless driving force for mass transfer. The transfer number presents the effects of the diffusional velocity. If B > 0 implies that the diffusional velocity is toward the bulk, while B < 0 indicates diffusion from the bulk toward the interface.

So, for the high mass transfer rate, the boundary condition at the surface becomes,

At y = 0 u=0, $v = (1+\sigma'^2)^{\frac{1}{2}} w_n$ and $\theta = \psi = 1$

The local and average Sherwood numbers can be derived as follows:
The local Sherwood number is defined as,

$$Sh = \frac{h_m l}{D} \tag{38}$$

Where h_m is the local mass transfer coefficient and is given by,

$$h_m = \frac{J_w}{C_w - C_\infty} \tag{39}$$

Where, J_m is the mass flux (diffusion) from the wall into the ambient and is defined as,

$$J_w = -D\frac{\partial C}{\partial n}\Big|_w \tag{40}$$

By substituting of equations (39), (40), and (33) in (38), the local Sherwood number becomes,

$$Sh = -\frac{Gr_c^{\frac{1}{4}}}{|R|^{\frac{1}{4}}(1-C_w)}\frac{(1+\sigma'^2)^{\frac{1}{2}}}{(4x)^{\frac{1}{4}}}\frac{\partial\psi}{\partial y}\Big|_{y=0} \tag{41}$$

Or $$Sh\left(\frac{4x}{Gr_c}\right)^{\frac{1}{4}}(1-C_w) = -\frac{1}{|R|^{\frac{1}{4}}}(1+\sigma'^2)^{\frac{1}{2}}\frac{\partial\psi}{\partial y}\Big|_{y=0} \tag{42}$$

And the average Sherwood number is

$$\overline{Sh} = -\frac{Gr_c^{\frac{1}{4}}}{s|R|^{\frac{1}{4}}(1-C_w)}\int_0^s \frac{(1+\sigma'^2)^{\frac{1}{2}}}{(4x)^{\frac{1}{4}}}\frac{\partial\psi}{\partial y}\Big|_{y=0} ds \tag{43}$$

Where, $$s = \int_0^x (1+\sigma'^2)^{\frac{1}{2}} dx \tag{44}$$

Which, is the length of the wavy surface from the leading edge to any point at the x-axis.

And $$ds = (1+\sigma'^2)^{\frac{1}{2}} dx \tag{45}$$

Therefore, the average Sherwood number can be written as,

$$\overline{Sh}\frac{(1-C_w)}{Gr_c^{\frac{1}{4}}} = -\frac{1}{s|R|^{\frac{1}{4}}}\int_0^x \frac{(1+\sigma'^2)^{\frac{1}{2}}}{(4x)^{\frac{1}{4}}}\frac{\partial\psi}{\partial y}\Big|_{y=0} dx \tag{46}$$

In a similar fashion, it can be shown that the local Nusselt number is

$$Nu = \frac{h\ell}{K} = -(1+\sigma'^2)^{\frac{1}{2}}\frac{Gr^{\frac{1}{4}}}{(4x)^{\frac{1}{4}}}\frac{\partial\theta}{\partial y}\Big|_{y=0} \tag{47}$$

Or $$Nu\left(\frac{4x}{Gr}\right)^{\frac{1}{4}} = -(1+\sigma'^2)^{\frac{1}{2}}\frac{\partial\theta}{\partial y}\Big|_{y=0} \tag{48}$$

Where h is the local heat transfer coefficient and K is the thermal conductivity.
And the average Nusselt number is

$$\frac{\overline{Nu}}{Gr^{\frac{1}{4}}} = -\frac{1}{s}\int_0^x \frac{(1+\sigma'^2)^{\frac{1}{2}}}{(4x)^{\frac{1}{4}}} \left.\frac{\partial\theta}{\partial y}\right|_{y=0} dx \tag{49}$$

RESULTS AND DISCUSSION

The governing equations are solved numerically using finite difference method. The computational grids are fitted to the body shape in (x,y) coordinates. The singularity at x=0 has been removed by the scaling, then the computation can be started at x=0 and marches down stream. The Newton Raphson method is used for linearizing the nonlinear terms and the Simpson rule is used for the numerical integration.

The numerical results are obtained for $y = \overline{x}(\overline{\sigma}) = \overline{\alpha}\sin k\overline{x}$ to study the geometric effect on the natural convection. The velocity component parallel to the x-axis, u, and the normal to x-axis, v, are shown in Figure.(2) and Figure (3), respectively, for α=0.1. The velocities profiles at the trough (x=1.75), and at the crest (x=2.25), are almost the same and can not be distinguished. The boundary layer is found to be thicker near the nodes than near the trough and the crest as shown in Figure (2).

In order to check the accuracy of the solution, the program was tested by reproducing other some available data in the literature. The obtained Nusselt number and Sherwood number were compared with the results obtained by Yao [1] and Bottemanne [19], which are confirmed experimentally by Rahman (10), and they are found in good agreement.

The temperature profiles along the y-direction are shown in Figure (4). Similar to the velocities profiles, the temperature profile difference between two nodes and between the trough and the crest is indistinguishable. The thermal boundary layer thickness near the node is larger than that near the trough and the crest. The mass fraction profile along y-direction is shown in Figure (5). Behavior similar to that of temperature profile is obtained.

Because the y-axis is normal to the x-axis and not to the wavy surface, the temperature gradient and mass fraction gradient are corrected before the heat transfer rate and mass transfer rate are calculated.

The effect of Prandtl number on the local heat transfer rate is shown in Figure (6). The results show that larger Nusselt numbers are associated with larger Prandtl numbers. The reason for this is that a larger Prandtl number corresponds to a thinner thermal boundary layer thickness relative to the flow boundary layer thickness, thereby resulting in a larger temperature gradient at the wall which in turn enhances the heat transfer rate.

The effect of Schmidt number, Sc, on the local Nusselt number for a given Prandtl number (Pr=0.72) is shown in Figure (7). Larger increase in Nusselt number occurs at smaller Schmidt numbers for because of the influence of the diffusion coefficient on the flow field and the thermal field. A higher Schmidt number corresponds to a smaller binary diffusion coefficient exerts a smaller influence on the flow field and the thermal field.

The heat transfer rate is enhanced for fluids with a higher Prandtl number. However, the Nusselt number is seen to decrease as the Schmidt number increases.

Figure (8) is showing that larger local Sherwood numbers are always associated with larger Schmidt numbers for any given value of R. The reason is that a large Schmidt number corresponds to a small binary diffusion coefficient, and hence a diffusion boundary layer thinner than momentum boundary layer is developed. As a result, a larger concentration gradient at the wall, this in turn enhances the mass transfer rate.

The effect of the amplitude of the wave on the local Nusselt and Sherwood numbers is shown in Figure (9). Both numbers are found to decrease as the amplitude increases. The rate of decreasing of Nusselt and Sherwood numbers are found to increase as the amplitude increases.

The effect of the amplitude on the Local Nusselt number increases by increasing Prandtl number as presented in Figure (6). For very small Prandtl number, the amplitude of the wavy surface has no noticeable effect on the heat transfer rate. Reducing the Prandtl number increases the thermal boundary layer thickness and as a result the effect of the amplitude of the wavy surface on the heat transfer rate is diminished.

Similar behavior is observed for the effect of the amplitude of the wavy surface on the local Sherwood number with for different values of Schmidt numbers as demonstrated in Figure (8). In this case, low Schmidt number corresponds to thick diffusion boundary layer in which the amplitude of the wavy surface has no noticeable effect on the mass transfer rate.

Figure (10) and Figure (11), are showing that for specific value of **α,** Pr=0.72, Sc=0.6 and R=0, the magnitude of the local heat transfer rate and mass transfer rate depend on the slope of the wavy surface. Near the trough and crest, where the gravitational force is parallel to the wavy surface, the velocity is larger and so are the heat transfer rate and mass transfer rate. The magnitude of the variation of the transfer rate decreases downstream because of the growing of the natural convection boundary layer.

The wavy variation of the average Nusselt number and Sherwood number can be observed only near the leading edge and gradually disappears downstream as shown in Figure (12) and Figure (13), respectively.

CONCLUSIONS

Natural Convection arising from the combined buoyancy force effects of thermal and species diffusion along a vertical sinusoidal wavy surface of a uniform wall temperature and uniform mass fraction is studied numerically. The local Nusselt number is found to increase when the buoyancy force from species diffusion assists the thermal buoyancy force and to decrease when it opposes the thermal buoyancy force.

Small Schmidt number enhances the heat transfer rate, while large Schmidt number is responsible for the increase of the mass transfer rate. Increasing the amplitude of the wavy surface is found to decrease the heat and mass transfer rates.

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NOMENCLATURES

a amplitude of the wave surface, (m)
C mass fraction of the diffusing component (Air)
D binary diffusion coefficient, [m^2/sec].
g gravitational acceleration, (m/sec^2).
Gr Thermal Grashof number
$Gr_{,c}$ Grashof number for mass diffusion.
ℓ Wave length
n Normal victor.
Nu Nusselt number
Pr Prandtl number
R Ratio of Grashof numbers (buoyancy ratio).
t Tangent victor.
S distance measured along the wavy surface from the leading edge, (m)
Sc Schmidt number.
Sh Sherwood number.
T Temperature [°C]
u, v Axial and normal velocity, (m/sec).
x, y Vertical and horizontal coordinates.

GREEK SYMBOLS

α Dimensionless amplitude of the wave ratio = a/L.
β Volumetric coefficient of thermal expansion.
β^* Volumetric coefficient of expansion with mass fraction,
σ Surface geometry function
ν Kinematic viscocity, (m2/sec).
μ Dynamic viscosity of the fluid (kg/m.sec)
ρ Density of the fluid (kg/m3)

SUPERSCRIPTS

$\bar{}$ Dimensional quantity.
^ Dimensionless quantity.
′ Derivation with respect to *x*.

SUBSCRIPTS

w condition at the surface (wall).
∞ condition at the free stream.

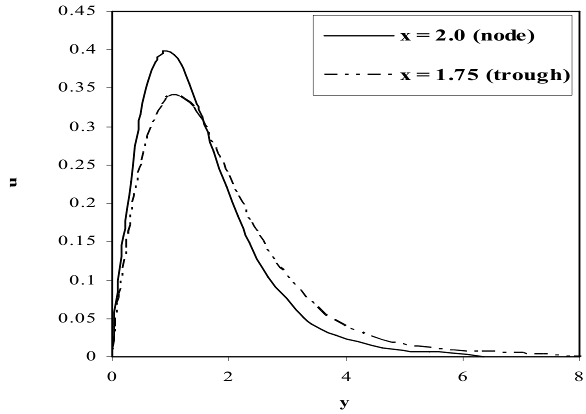

Figure 2: axial velocity profile
(α = 0.1, Pr =0.72, Sc =0.6, R=1.0, B=0.0)

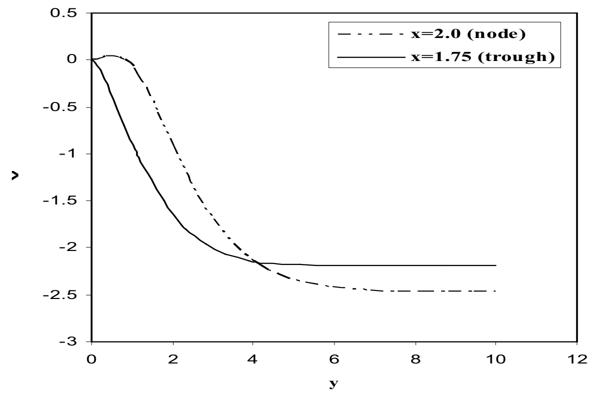

Figure 3 : Normal velocity profile
(α = 0.1, Pr = 0.72, Sc = 0.6, R = 1.0, B = 0.0)

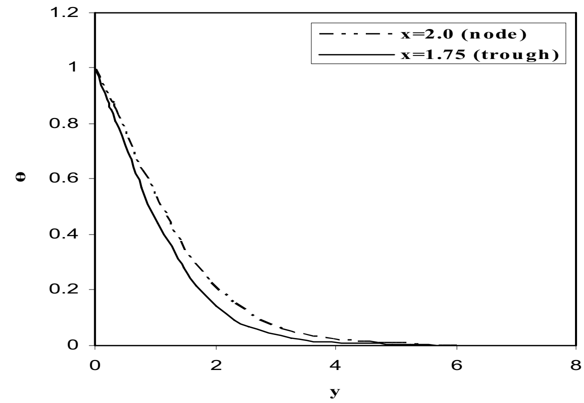

Figure 4 : Temperature profile
(α = 0.1, Pr =0.72, R= 1.0, B = 0.0)

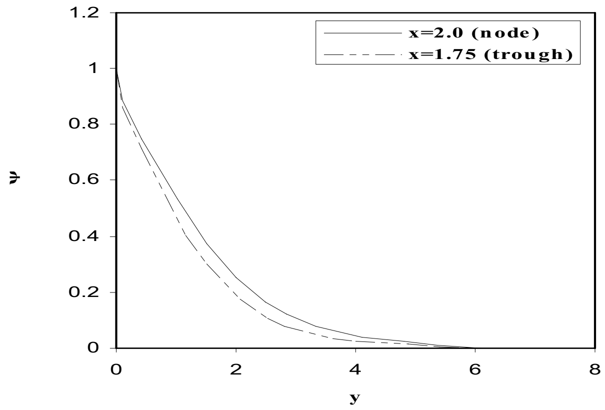

Figure 5 : Mass fraction profile
(α = 0.1, Pr = 0.72,Sc = 0.6, R = 1.0, B = 0.0)

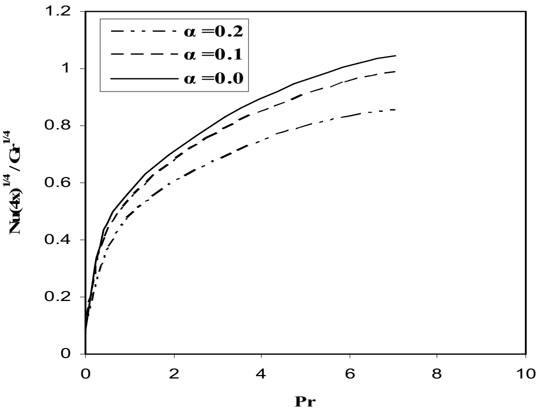

Figure 6 : Effect of Pr on the local heat transfer rate
(R = 0, B = 0)

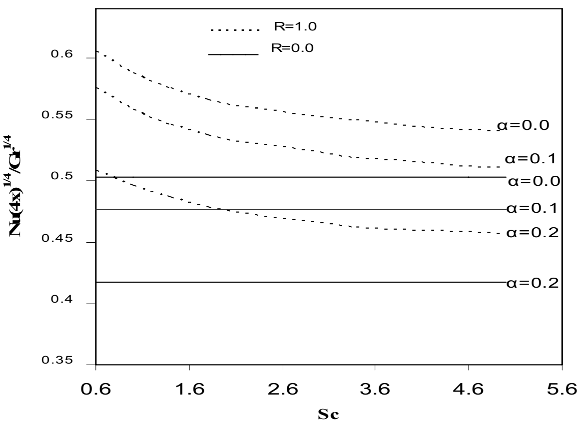

Figure 7 : Effect of Schmidt number on the local heat transfer rate
(x=1.0, Pr= 0.72, B=0.0)

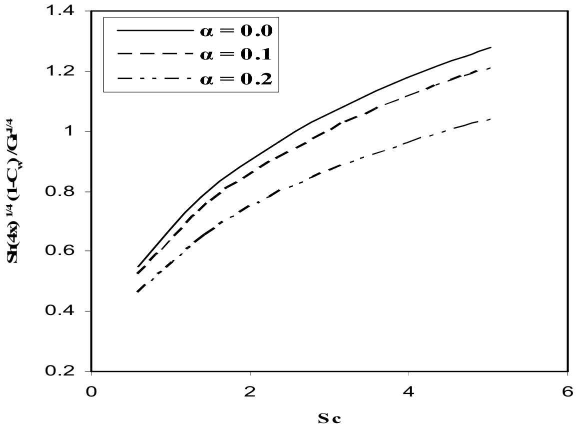

Figure 8 : Effect of Sc on the local mass transfer rate (x=1.0, Pr =0.72, R = 1.0)

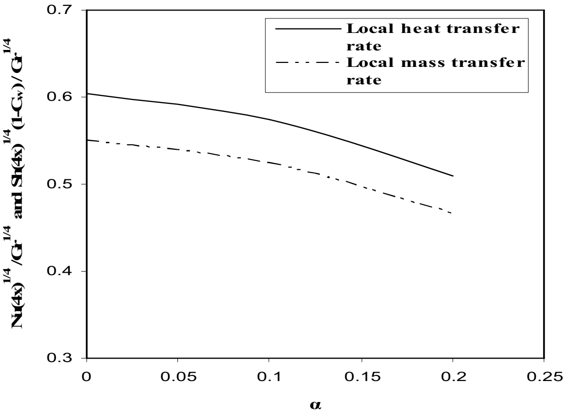

Figure 9 : Effect of α on the local heat and mass transfer rates (x= 1.0, Pr = 0.72, Sc = 0.6, R = 1.0, B = 0.0)