USING THE THREE EQUATION METHOD FOR CONCRETE MIX DESIGN

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الملخص

يعتبر التصميم الجيد للخلطات الخرسانية من الركائز الأساسية لتنفيذ أي عضو أو منشأ خرساني كفؤ قادر على تحقيق المتطلبات الاقتصادية والخدمية ومتطلبات المتانة والآمان والكفاءة طول مدة العمر الوظيفي للعضو أو المبنى. والمقصود بتصميم الخلطة الخرسانية هو تحديد كميات المواد الأساسية الداخلة في تكوين الخرسانة (الإسمنت، الركام الخشِن، الركام الناعم والماء). ويوجد في الوقت الحاضر العديد من الطرق العالمية والمعتمدة محليا في تصميم الخلطات مثل: طريقة المعهد الأمريكي للخرسانة (ACI)، والطريقة البريطانية (BS)، وتلك الطرق تستخدم على نطاق واسع بالجماهيرية في تصميم الخلطات الخرسانية لدى كل من المراكز البحثية والجامعات والخلاطات المركزية للخرسانة وكذلك مصانع إنتاج الأعضاء الخرسانية سابقة الصب (مثل مصانع الكمرات الخرسانية سابقة الإجهاد ومصانع الأعمدة الخرسانية وأنابيب الخرسانة.....الخ). وتعتمد تلك الطرق على بعض المعادلات والعلاقات البيانية الناتجة من التحليل الرياضي للنتائج المتحصل عليها من خبرات سابقة في هذا المجال. وعلى الغالب فإن طرق تصميم الخلطات تعطي مؤشرات وخطوط عريضة للمصمم يتم التحقق منها وتعديلها بإجراء خلطات تجريبية في المعامل المحلية لكي تحقق المتغيرات المتعلقة بخواص المواد المحلية المستعملة والظروف البيئية المحيطة.

في هده الورقة سيتم عرض الخطوات المتبعة لتصميم الخلطات الخرسانية بطريقة الثلاثة معادلات والمعروفة بطريقة (Bolomeya) وهي طريقة متبعة لدى بعض المراكز البحثية بدولة بولندا والتي تم تطبيقها حديثا بمعامل قسم الهندسة المدنية بجامعة الفاتح.النتائج المتحصل عليها بهذه الطريقة باستخدام المواد والظروف البيئية المحلية.

ABSTRACT

A good mixing design for concrete mixes is considered as a milestone for the construction of any concrete member or structure capable to acquire economical, service and durability requirements, as well as safety and efficiency along its functional life. The definition of a mix design is to determine the quantities of the main components of concrete, namely; cement, aggregate and water to mix one cubic meter of concrete.

Currently, there are many international methods locally approved for mix designs, such as: The American concrete institute method (ACI) and the British standard method (BS), which are widely used in Libya at research centers, universities, concrete batch plants as well as pre-cast concrete manufacturing plants (e.g.: pre-stressed concrete beams, concrete columns and slabs, ……etc). These methods depend on certain equations and graphs resulted from mathematical analysis for the results obtained from previous experience in this field. Generally speaking, mix design methods give some indications and start points, and designers need to test and adjust the

obtained results by trial mixes in the local laboratories in order to investigate the variables related to the characteristics and properties of the local materials and the effect of the surrounding environment conditions.

This paper presents::

- The steps used for mix design using the three equations method, which is so-called Bolomeya method, and is a method used in some research centers in Poland and was recently applied in the laboratories of the civil engineering department in the University of Al-Fateh.
- Results obtained by this method using the local materials subject to local environmental conditions.

KEYWORDS: Bolomeya method; Mix design; Concrete mixes; Trial mixes; Sterna table.

INTRODUCTION

The wide spread of concrete usage in different engineering applications was accompanied with many researches and studies during the past four decades. This was resulted in more understanding of the concrete composition and behavior leading to a comprehensive development in the concrete technology and quality control. Because the concrete behavior, whether fresh or hardened concrete, depends basically and directly on the behavior of its components, ratios and relationship between these materials, thus obtaining a concrete with certain properties depends on the concrete mix design. Concrete mix design includes generally two main steps:

1. Selection of the main components suitable for the concrete (cement, aggregate, water, and additives);
2. Determination of the quantities in ratios in more economical ways accomplishing the workability, strength and efficiency requirements.

Currently, there are many international methods locally approved for mix designs, although they are all related directly to each other, but they give relatively the same quantities of the mix components, and they all are capable to give good concrete mix [1]. It is important to consider that these methods give approximate quantities which should be checked by experimental mixes in order to obtain results suitable for the requirements of the local environment and local materials used in more economical and efficient ways. ACI method and BS method are considered the most common used methods. Both of these methods depend on graphs and standard tables resulted from experience and researches on concrete mix design and production as well as studying the properties of the materials used [2]. Along with the aforementioned methods, there are many methods used for concrete mix design, such as what is so-called " The Three Equations Method (Bolomeya Method) ", which will be illustrated in details in this paper, in addition to the assessment of the results of concrete mixes produced by this method [3].

CONCRETE MIX DESIGN USING THE THREE EQUATIONS METHOD

The procedure for using this method for concrete mix design is as the following:

1. Determination of the compressive strength of the concrete mix to be designed (field strength) [fc` (req.)];
2. Calculation of the concrete compressive strength in the laboratory taking into consideration that it is more than the field strength by 30% in average [fc` (lab.)]

$fc`$ (lab.) = 1.3 * $fc`$ (req.) (1)

Note: The ratio between the field strength and the laboratory strength depends on many factors, such as accuracy of the equipment, experience, size of the samples,.etc.

3. Testing the degree of workability. This factor is determined in the field or in the laboratory using many methods such as Slump Cone. The workability depends on many factors such as density of reinforcement, maximum size of coarse aggregate, casting and method of compaction, etc;
4. Determination of cement- water ratio (C/W) using the First Equation of Design known as the First Bolomeya Equation, as following:

$$\frac{C}{W} = \left[\frac{fc'}{A_1} + 0.5\right] \quad \text{if} \quad (C/W) < 2.5 \qquad (2\text{–a})$$

$$\frac{C}{W} = \left[\frac{fc'}{A_2} - 0.5\right] \quad \text{if} \quad (C/W) \geq 2.5 \qquad (2\text{–b})$$

Where C/W is the cement- water ratio, $fc`$ is the laboratory compressive strength of concrete, A_1 and A_2 are variables depend on shape of coarse aggregate used and the compressive strength of cement. Note that also:

- The values of A_1 and A_2 are determined using tables as shown in Table (1).
- The value of A_1 is substituted in equation (2-a); if the result is less than 2.5 then the value is obtained, and if the value is greater than or equal to 2.5, then the W/C ratio is calculated by using equation (2-b).

5. The required amount of water for the concrete mix (W_{Total}) is calculated using the Second Equation of Design, as follows:

W_{Total} = C*(Wc) + A*(Wa) (3)

Where:

W_{Total}: Quantity of water required for the concrete mix (liter/m^3).
A: Weight of fine and coarse aggregate required for the concrete mix (kg/m^3).
C: Weight of cement required for the concrete mix (kg/m^3).
Wc: Quantity of water absorbed by one kilogram of cement, to be determined by the standard consistency of cement (%).
Wa: Quantity of water absorbed by one kilogram of aggregate (%).

The required quantity of water depends on the workability, type and size of coarse aggregate, grading of fine and coarse aggregate, separated and mixed, and is to be calculated using special tables linking between the grading of aggregate with the absorption ratios, known as Sterna Table [3], as shown in Table (2).

6. Calculating the volume of the component materials of the mix using the volume equation, which is the Third Equation of Design, as follows:

$$\frac{W}{1000}+\frac{A}{1000\rho_a}+\frac{C}{1000\rho_C}=1 \quad (4)$$

Where:

C: Cement weight (kg/m^3)
W Water weight (kg/m^3)
A: Total Aggregate weight (kg/m^3)
ρ_c: Specific gravity of cement
ρ_a: Specific gravity of aggregate

7. Solving the three equations of design (2, 3, and 4) for W, C, and A in order to obtain the required quantities of the components for the mix design according to the specified conditions.

Table 1: Calculation of A1 and A2 values [3]

Aggregate shape	**Variable A**	**Compressive strength of cement after 28 days** (N/mm^2)		
		32.5	**42.5**	**52.5**
Round	A1	18.0	20.0	21.0
	A2	12.0	13.0	14.5
Angular	A1	20.0	22.0	24.0
	A2	13.5	14.5	16.0

Table 2: Water quantity to be absorbed by one kilogram of aggregate (Wa) and cement (Wc) according to the workability (Sterna table) [3]

Sieve size (mm)	Workability		
	Very low	Medium	Very high
37/19	0.011**	0.013	0.016
19/14	0.014	0.016	0.022
14/10	0.017	0.02	0.027
10/5	0.022	0.026	0.034
5/2.36	0.028	0.032	0.044
2.36/1.18	0.037	0.043	0.058
1.18/0.6	0.05	0.058	0.077
0.6/0.3	0.072	0.084	0.112
0.3/0.15	0.104	0.122	0.131
0.15/0	0.205	0.239	0.296
Wc	0.271*	0.273	0.275

* Quantity of water absorbed by one kilogram of cement (Wc)
** Quantity of water absorbed by one kilogram of aggregate (Wa)

EXPERIMENTAL

This part represents the design and production of a number of concrete mixes following the Three Equations Method and using the local raw materials. This paper illustrates the results obtained by production of four concrete mixes with design compressive strengths of 15, 20, 25, and 30 MPa, where the water-cement ratio (W/C) is 0.69, 0.56, 0.48, and 0.42, respectively, and the workability is tested for low, medium and high degrees. These mixes were prepared and produced in the concrete laboratory at the Faculty of Engineering – Al-fateh University - in researches carried on by students as a part of the academic requirements for B.Sc. Degree in Civil Engineering [4, 5]

These concrete mixes were produced following four main stages summarized as follows:

Materials used in the concrete mix and quality control

I: Cement

The cement used is the Ordinary Portland Cement supplied by Zliten Factory for Cement [6].

II: Coarse aggregate

The coarse aggregate used in the experiment was angular aggregate with maximum size of 19 mm, imported from a quarry near Tarhuna (nearly 60 km south of Tripoli city). Tests are carried out to check the aggregate specifications according to the BS 812-1992, [7].

III: Fine aggregate

Natural, fine aggregate that was used in the mixture was natural beach sand from Zlietn quarry (nearly 200 km east of Tripoli city). The fine sand was used with particular size not exceeding 2 mm, [7].

IV: Mixing water

Fresh, free of dirt and organic materials water was used, with a percentage of total dissolved salts not exceeding 2000 particles per million as per Libyan standards.

Samples

Standard cylinders of 150 mm diameter and 300 mm height were used as samples for compressive strength and tensile strength of concrete. A total of 72 samples were cast in average of 6 samples per mix (15, 20, 25, and 30 N/mm^2) and a total of 24 samples per each degree of workability (very low, medium, and high).

Method of concrete mix design and calculation of mix components

The objective of a concrete mix design is to determine the weights of the main components of a concrete mix according to certain requirements, such as strength, workability, etc. In this paper, the Three Equations Method (Bolomeya method) [3] is used for the mix design, which was previously illustrated. A numerical example for a mix design of concrete with a compressive strength of 20 N/mm^2 and low workability (slump = zero) is shown here.

1. $fc`$ (lab.) = 1.3 * $fc`$ (req.)

   $fc`$ (lab.) = 1.3 * 20 = 26 (MPa)

2. Calculate the water-cement ratio (W/C) knowing that the compressive strength of cement is (32.5 N/mm^2) and the aggregate shape is angular, so the values of A1 and A2 from Table (1) are 20 and 13.5, respectively. Substituting in equation (2-a) we obtain:

$$\frac{C}{W} = \left[\frac{26}{20} + 0.5\right] = 1.8 \qquad (5)$$

C = 1.8 W First Equation of Design

As the obtained value is less than 2.5, there is no need to use equation (2-b).

3. Calculate the water quantity required for the mix knowing the degree of workability, shape of aggregate, size of the fine and coarse aggregate using Sterna table (See table 1 and 2), as follows:
   - Determine the amount of water absorbed by one kilogram of cement (Wc) knowing the standard consistency of cement and the workability, which is **0.271** in this example;
   - Knowing the grading of sand and aggregate and the percentage of their mixture, prepare a table format as shown in Table 3 to obtain the amount of water absorbed by one kilogram of coarse aggregate (Wa), which is **0.0488** in this example:

$W_{(Total)} = 0.271 * C + 0.0488 * A$ Second Equation of Design

Table 3: Calculation of water absorbed by aggregate (Wa).

Sieve Size (mm)	Sieve Analysis of Aggregate				Sieve Analysis of Combined Aggregate				Wa	Wa* %
	Fine Agg.		Coarse Agg.		40% Fine Agg.	60% Coarse Agg.				
	Retained (%)	Passed (%)	Retained (%)	Passed (%)	Retained (%)	Retained (%)	Retained (%)	Passed (%)	Very low Workability	
37/19			2.2	100		1.32	1.320	100	0.011	0.0145
19/14			18.3	97.8		10.98	10.980	98.68	0.014	0.1537
14/10			45.5	79.5		27.3	27.300	87.70	0.017	0.4641
10/5			33.77	34		20.262	20.262	60.40	0.022	0.4458
5/2.36			0.23	0.23		0.138	0.138	40.14	0.028	0.0039
2.36/1.18	0.28	100			0.112		0.112	40.00	0.037	0.0041
1.18/0.6	0.76	99.72			0.304		0.304	39.89	0.050	0.0152
0.6/0.3	35.92	98.96			14.368		14.368	39.58	0.072	1.0345
0.3/0.15	60.04	63.04			24.016		24.016	25.22	0.104	2.4977
0.15/0	3	3			1.2		1.200	1.20	0.205	0.246
Total	100		100		40	60	100			4.8795
									Wa	0.0488

* (Wa (%) = [Wa] x [Combined Retained (%)]

4. The Third Equation of Design is to be obtained by substituting in the volume equation:

$[C/3.15] + [A/2.65] + W = 1000$ Third Equation of Design (6)

Solving the three equations, we obtain the weights of the different components of the concrete mix, as follows: Cement = 311.35 kg/m^3, Water = 177.91 kg/m^3, Aggregate = 1916.67 kg/m^3. Table (4): shows the weights of the concrete mix components.

Sample curing

After casting, the samples were covered with a plastic film to avoid evaporation and plastic shrinkage, and are to be left 24 hours in the laboratory environment. Samples were to be marked to distinguish between them and were submersed in water in room temperature for 28 days then were to be tested for compressive strength and tensile strength of concrete.

Table 4: Weights of the concrete mix components

Design Compressive strength (N/mm^2)	Water-Cement ratio (W/C)	Mix components * (kg/m^3)	Workability		
			Low	Medium	High
19.5	0.69	C	197.92	231.77	305.10
		W	136.49	159.84	210.41
		Ac	1249.05	1195.87	1080.67
		Af	832.69	797.24	720.45
26	0.56	C	264.51	311.79	415.42
		W	149.44	176.15	234.70
		Ac	1195.88	1130.78	988.20
		Af	797.25	753.86	658.79
32.5	0.48	C	351.74	410.01	554.50
		W	168.30	196.13	265.30
		Ac	1153.50	1050.91	871.50
		Af	769.00	700.60	581.00
39	0.42	C	440.98	526.10	728.83
		W	183.74	220.45	303.68
		Ac	1054.97	954.10	725.32
		Af	703.32	636.10	483.50

* Cement (C), Water (W), Coarse aggregate (Ac), Fine aggregate (A_f).

Laboratory tests

a. Slump test

This test is used to determine workability degree of the concrete mix in order to investigate the consistency of concrete and to check the design workability according to the British Standards BS 1881-Part 102 [8]. Table (5) shows the results of the different mixes used for the different workability degrees.

b. Compressive strength test

The objective of this test is to determine the maximum compressive strength of the hardened concrete subject to compressive stresses. It is carried out by putting the samples under a compression on the centerline of the concrete samples used. The load is increased gradually up to failure. In order to assure equilibrium distribution of loads on the sample surface, the surfaces were treated using sulphate according to the American Standards ASTM C617-87 [9]. The compressive strength is calculated as the mean of three samples per each mix, using the following formula:

$$f'_c = P/A$$

Where:

P: failure load, *N*

A: cross sectional area, sq. mm.

Figure (1) shows the results of compressive strength for the mixes used for different degrees of workability.

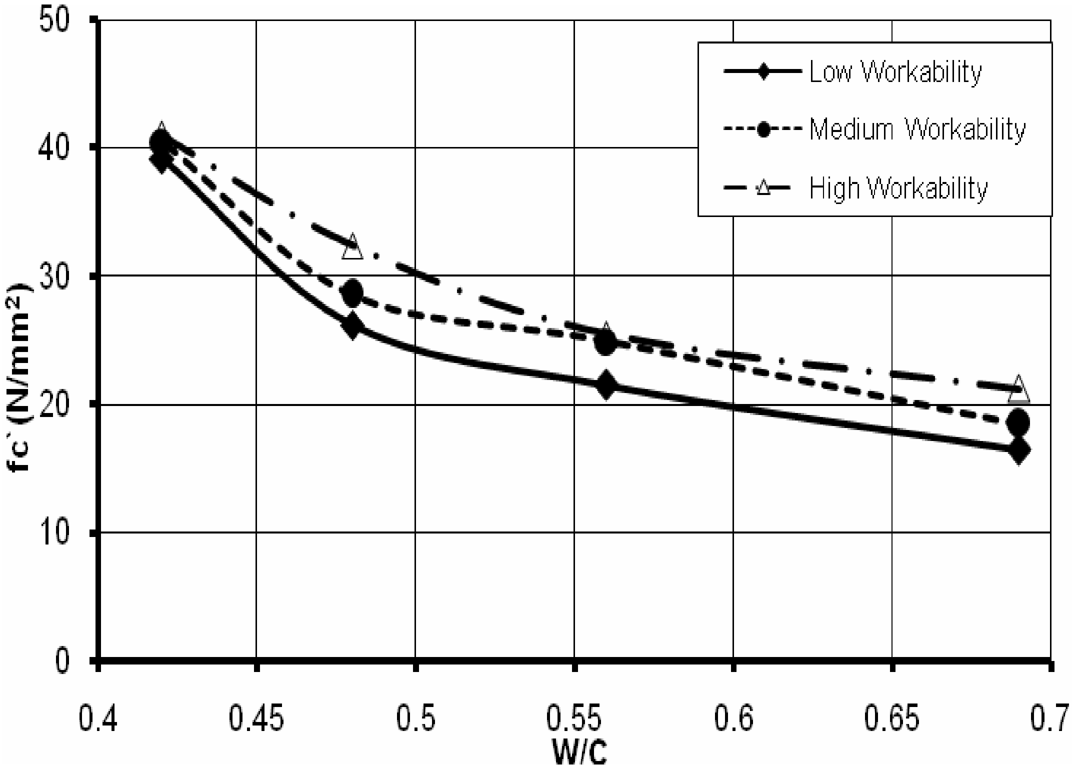

Figure 1: Test results for compressive strength of concrete samples

c. Tensile strength test

The objective of this test is to determine the tensile strength of concrete indirectly, by exposing the samples into a vertical pressure on the centerline of the sample. The load is applied on a small area of the cylinder circumference by two metal rods and a small plate of plywood. The test done according to the American Standard ASTM –C496-86 [10]. The tensile strength is calculated by the mean of three samples per each mix, using the following formula:

$$f_t = 2P/\pi DL$$

Where:

P: failure load, *N;* D: diameter of the cylinder, mm; L: length of the sample, mm

Table (5) summarizes the results of the tensile strength for the mixes used for different degrees of workability.

Table 5: Results of slump test, compressive strength and tensile strength of concrete mixes

Design Compressive strength* (N/mm^2)	Water-Cement ratio (W/C)	Degree of Workability	Slump (mm)	Computed Compressive strength (N/mm^2)	Computed Tensile strength (N/mm^2)
19.5	0.69	Low	60	16.5	1.90
		Medium	100	18.5	1.82
		High	150	21.2	2.03
26	0.56	Low	80	21.5	1.96
		Medium	130	24.9	2.03
		High	160	25.5	2.26
32.5	0.48	Low	100	26.2	2.88
		Medium	150	28.6	3.07
		High	210	32.4	2.12
39	0.42	Low	100	39.2	3.53
		Medium	180	40.5	2.68
		High	280	41.0	3.95

*fc\`(Design) ≥ $fc$\`(Computed) ≥ fc\`(Required)

DISCUSSION

The results obtained from calculations carried out for the concrete mix components and quality control tests are showed that:

1. The concrete mix components obtained using this method is in compliance with those could be obtained by the common design methods such as ACI and BS methods;
2. The results of the slump test is relatively less than by 5 to 10 percent in average from the common limits of workability at each degree of workability;
3. This method is efficiently accomplishing the requirements of the compressive and tensile strengths.

CONCLUSIONS AND RECOMMENDATIONS

1. This method is considered as one of the important introductions to the concrete mix design methods due to the following advantages:
   - Introducing the effect of water absorption by cement in the design process;
   - Introducing the effect of water absorption by fine and coarse aggregate as well as the grading in the design.
2. In order to make this method more efficient, it is recommended to carry out more researches on tables to give the quantities of water absorbed by the cement and aggregate for the local raw materials similar to those used in this method (Sterna Tables).

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