RESPONSE OF AIRCRAFT WING AS MULTIBODY SYSTEM NEAR GROUND USING NATURAL AND JOINT CO-ORDINATES

Saad M. Issa and Abdulhamid A. Ghmmam

Department of Aeronautical Engineering, Faculty of Engineering
Tripoli University, Libya
E-mail:- sissa@aerodept.edu.ly

الملخص

جناح الطائرة كنظام متعدد الأجسام سوف يتم اختباره حسابيا باستعمال الإحداثيات الطبيعية والنسبية. مرونة الجناح تم اعتبارها عن طريق تقسيم الجناح إلى أجسام صغيرة متصلة بمفاصل مرنة. هذه المفاصل المرنة متكونة من لوالب التوائيه ومفاصل ميكانيكية والصلابة لتلك اللوالب تمثل المرونة للنظام. القوى الهوائية الغير ثابتة والمؤثرة على الجناح تم احتسابها وتضمينها في النموذج تحت الاختبار.

معدلات الحركة المتحصل عليها تم استنتاجها باستخدام طريقة القدرة الافتراضية بواسطة الإحداثيات الطبيعية تم بعد ذلك تحويلها إلى الإحداثيات النسبية. لاختبار والتحقق من هذه الطريقة الجديدة تم إيجاد استجابة جناح الطائرة كنظام متعدد الأجسام بالقرب وبعيد عن الأرض لأول ستة أشكال للنسق.

ABSTRACT

Aircraft wing as multibody system based on natural and joint co-ordinates is numerically investigated. The flexibility of the system's (wing) is being considered through discretizing the wing into a series of small rigid bodies interconnected by elastic joints. These elastic joints consist of flexural springs and mechanical joints. The stiffness of springs represents the elastic behaviour of the system's bodies. The unsteady air loads acting on the wing are evaluated by the use of Unsteady Vortex Lattice Method. The equations of motion obtained, through using of the principal of virtual power in terms of the natural co-ordinates, are transformed into another set of equations using velocity transformations. To asses and validate this new approach, the aircraft wing response near and far out of ground is obtained in terms of its first six mode shapes.

KEYWARDS: Aircraft wing; Multibody system; Unsteady aerodynamics; Natural and joint coordinates; Equation of motion; Mode shapes.

INTRODUCTION

The multibody system (MBS) is a collection of two or more bodies interconnected together either directly by joints or indirectly by other mechanical elements like springs, shock absorbers or dampers. The derivation of the equations of motion for computational multibody dynamics has been the topic of many research activities [1-4]. During recent years there have been many attempts to develop efficient methods for obtaining equations of motion for MBS. Most of these attempts have been motivated by advances in computer hardware and software as well as by advances in numerical methods and in the formulation of the equations of motion.

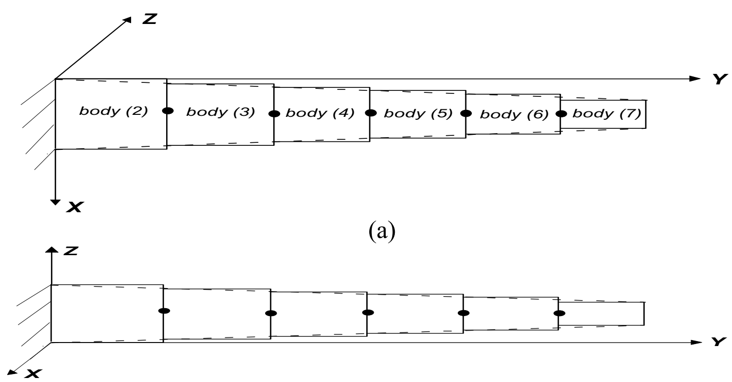
The aircraft wing is an airborne structure and it can be an example of MBS, consisting of rigid and flexible bodies subjected to different dynamic loading conditions. Here, the aircraft wing is considered to be MBS consisting of rigid bodies interconnected only by kinematic joints. Instead of using Euler angles or Euler parameters to define the spatial orientation of each rigid body, a system of natural co-ordinates is used. In such system, Cartesian coordinates of two or more points and Cartesian component of one or more unit vectors are rigidly attached to the body. These points and vectors describe the translation and rotation motions of the system, respectively.

The objective of this paper is to present a mathematical model of aircraft wing based on the natural and joint coordinates with aerodynamic loads taken into account. The derived resulting equations of the motion of the system will be obtained in terms of natural coordinates through using of virtual power formulation, and transformed into joint coordinates using velocity transformation process. A mathematical model of aircraft wing MBS based on this formulation will be developed and equations will be solved numerically.

WING STRUCTURAL MODELLING AND CASE STUDY

Wing Structural modelling:

The wing (right and left) is modelled as two identical flexible cantilevers. It is divided into a series of small (box) rigid bodies interconnected by elastic joints. These elastic joints consist of mechanical joints (universal joint) and flexural springs. The system flexibility is represented by these flexural springs. The type of joint is chosen, to allow a bending and torsion motions of each body. Each semi-wing is divided spanwise into six small boxes; each sectional wing in the system represents a body. Finally all the bodies of the descretized wing model form a part of the aircraft MBS under consideration. Figure (1) shows wing bodies of aircraft right wing.



(b)

Figure 1: A right wing rigid bodies system (where (•), represents the elastic joints) (a) top view (b) side view

Natural coordinates are used as dependent coordinates to specify the system configuration and to describe the motion of the system of bodies. A universal joint is used in line with a flexural spring to connect every two contiguous bodies except the ones (body 2) which are rigidly connected. There is no rotational motion between the wing and fuselage (body 1). Each body is assumed to be described by two basic points and two unit vectors except the ones connected to fuselage which are described by two points and one unit vector. A body fixed frame of coordinate of each body is attached to the joint point which is connected and shared by the contiguous bodies.

Case Study:

The aircraft wing has the following characteristics: root chord = 7.7 m, wing aspect ratio = 7.2, taper ratio = 0.30, the wing thickness at root is equal to 0.85 m, and at wing tip is 0.26 m, the wing skin thickness is 0.0234 m. The wing span is equal to 32.92 m, i.e. the length of each body in spanwise direction, is equal to 2.74 m. The wing height above the ground is 2.7 m. The cross section of body 2 is shown in Figure (2). Other bodies have the same cross section with different dimensions. The wing data are assured to represent a hypothetical wing rather very close to real one.

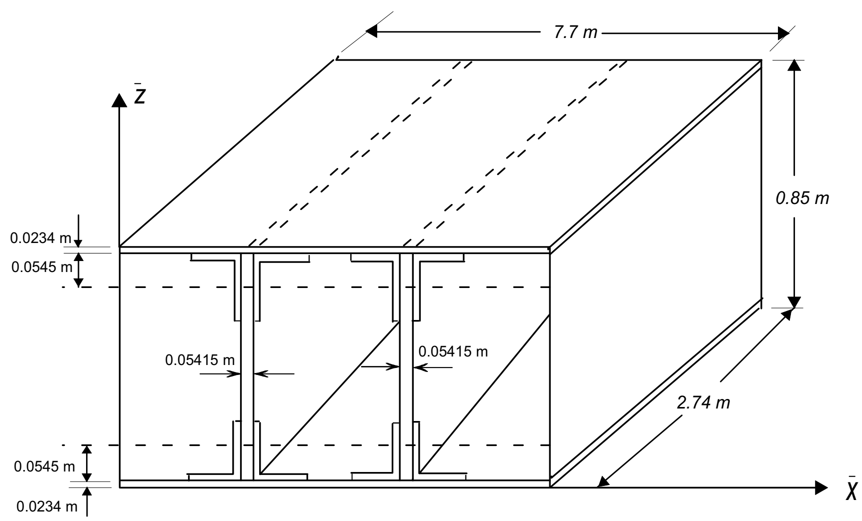


Figure 2: Wing's body cross section

Since both sides of wing are identical, the inertia properties of only right wing model are presented in Table (1) for all six bodies. We have to point out that, these inertia properties are calculated with respect to each body fixed frame of reference attached to each body, the mass products of inertia are equal to zero, because of symmetry of aircraft axes.

Table 1: Inertia properties of wing bodies

Mass Moment of Inertia (kg.m^2)			Mass (kg)	Body Number
I_{xx}	I_{yy}	I_{zz}		
12542	8427	5354	1685	2
11767	7907	5002	1581	3
11054	7295	4635	1459	4
10130	6685	4277	1337	5
9206	6075	3860	1215	6
8289	5470	3475	1094	7

AERODYNAMIC MODEL

The aircraft wing is subjected to many different dynamic loads among them are aerodynamic loads. In this study, only lift and drag forces as well as pitching moment, are considered. The aerodynamic loads are calculated for each small body of the wing. The evaluation of the aerodynamic Loads (lift, drag and moment) is all based on the proper integration of the pressure on the lifting surface (wing). The flow is assumed to be inviscid, irrotational and incompressible. In this paper the pressure is obtained through an integral representation based on the potential model with ground effect being taken into account. Figure (3) shows schematically wing plan form, which is divided into panels, vortex rings are used as singularity elements for the wing, its image and its wake.

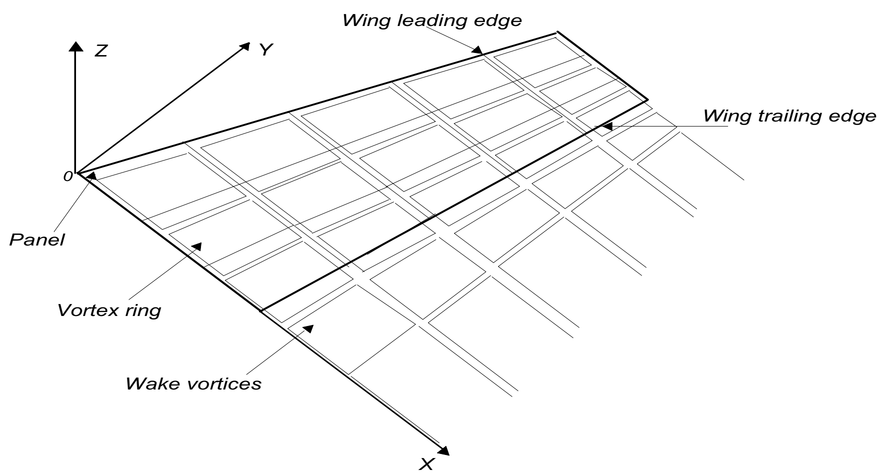


Figure 3: Panels and vortex rings model for a thin lifting surface

The pressure P can be determined from the unsteady Bernoulli equation as:

$$\frac{P_\infty - P}{\rho} = -\boldsymbol{V}^2 + \Phi_t \qquad (1)$$

where $\boldsymbol{V}^2 = \Phi_x^2 + \Phi_y^2 + \Phi_z^2$, $\Phi_t = \frac{\partial \Phi}{\partial t}$. Φ is the flow velocity potential and ρ is the air density. The velocity of the control point is computed according to the relationship for a body

$$\boldsymbol{V_f} = \boldsymbol{V_o} + \boldsymbol{\Omega} \times \boldsymbol{r} + \mathbf{v}_{ref} \qquad (2)$$

where $\boldsymbol{V_o}$ is the velocity of the origin of a moving reference frame (point 0) that is attached to the wing, $\boldsymbol{\Omega}$ is the angular velocity of the moving reference frame and is assumed equal to zero, and r the position of the control point relative to the moving reference frame, and $\boldsymbol{v}_{ref} = (\dot{x}, \dot{y}, \dot{z})$. Figure (4) shows a schematic representation of wing and its image near ground.

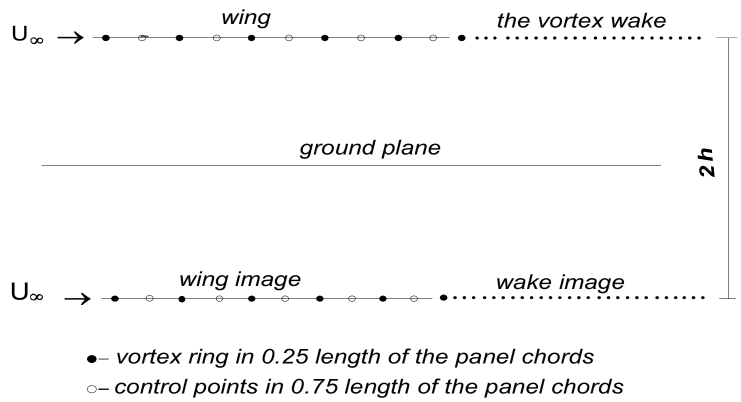


Figure 4: The wing and its image near the ground

Concluding; the steady state solution technique can be updated to treat unsteady flows. The first step is to compute the influence matrix $\boldsymbol{A}_{ij}$, where $\boldsymbol{A}_{ij}$ is the normal component of the velocity at the control point of element *i* generated by the unit circulation around the vertex segments enclosing element *j* and its image. By applying the proper boundary conditions [5] we arrive to this equation:

$$\sum_{j=1}^{N} A_{ij}\Gamma_j = RHS_j \qquad (3)$$

where $\boldsymbol{A}_{ij} = [(\zeta,\eta)_{ij} + (\zeta,\eta)_{ij}^{image}].n_i$, $i = 1,2,....,N$, N being the number of elements. $\boldsymbol{A}_{ij}$ are the influence coefficients that represent the normal components of the velocities at the control points and Γ_j is the circulation around the vertex segments enclosing element *j*, and (*RHS*) is the right hand side vector. The *RHS* vector is:

$$RHS_J = -\left((\mu+\zeta_w)\sin\alpha + \eta_w\cos\alpha\right)_J \qquad (4)$$

Where **μ** is the body forward velocity, which is assumed to be the same for all the rigid bodies, and α is the angle of attack.

When the circulation distribution Γ_j after the solution of equation (3) is obtained, the difference in pressure across the lifting surface is computed at each control point. The pressure difference is defined as:

$$\nabla P = P_l - P_u = \rho\left[\left(\frac{V^2}{2}\right)_u - \left(\frac{V^2}{2}\right)_l + \left(\frac{\partial\Phi}{\partial t}\right)_u - \left(\frac{\partial\Phi}{\partial t}\right)_l\right] \qquad (5)$$

To obtain the loads, the difference of pressure is multiplied by the area of the panel to produce the force on the panel. The panel forces and their moments are added and the resultants are resolved into lift, drag, pitching moment etc. Therefore, the aerodynamic loads will be calculated and considered as external forces for each small body of the wing, and then these forces will be expressed in terms of natural coordinates and will be incorporated in the system equations of motion.

THE SYSTEM EQUATIONS OF MOTION

The resulting equations of motion are obtained through using of virtual power formulation, which are transformed into joint coordinates using velocity transformation. A mathematical model of wing MBS based on this formulation was developed in ref. [6], and is extended and applied to aircraft wing MBS under consideration.

THE VELOCITY TRANSFORMATION PROCESS

Since the aircraft wing MBS have a large number of bodies, the velocity transformation matrix for the whole wing system is big from dimensional point of view. Therefore, only the velocity transformation matrix of the right wing will be determined in detail.

The aircraft right wing shown in Figure (5) is considered. It consists of six bodies interconnected by universal joints B_3, B_4, B_5, B_6 and B_7, except the ones connected to the fuselage (body 2), where, the wing is rigidly connected to the fuselage. Therefore, there is no rotational motion between the wing and fuselage. Defining $\left(\theta_j^1, \theta_j^2\right)^T$, for j = 3,......,7, as a vector of joint coordinates, are relative rotational coordinates about universal joints. The relative rotational velocities $\left(\dot{\theta}_j^1, \dot{\theta}_j^2\right)^T$, for j = 3,......,7, are defined as joint velocities where the two superscripts 1 and 2 are referred to the two joint axes.

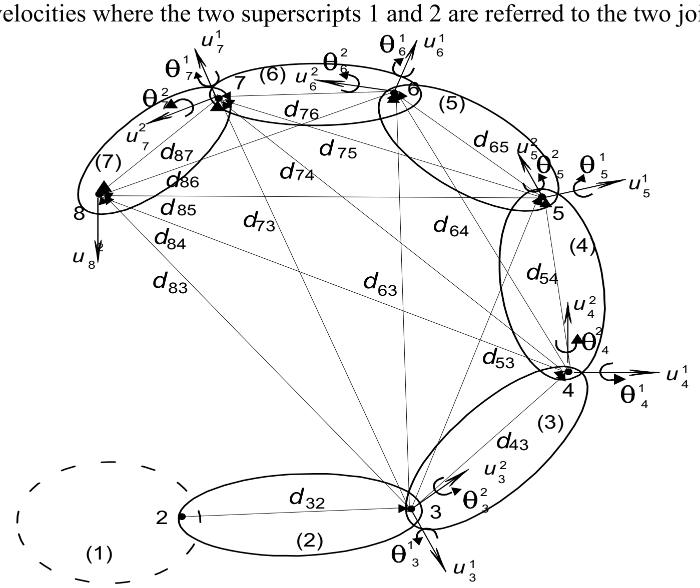


Figure 5: A schematic representation of vector d_{ij} for kinematic joints

Since the first body is rigidly connected to the fuselage, the basic point 2 is shared between the fuselage and body 2. The velocity of this basic point is dependent on the fuselage velocity. The basic points from 2 to 8, basic points 2, 3, 4, 5, 6, 7, are considered to be the reference points for bodies 2, 3, 4, 5, 6, 7, respectively.

Generally, the relative angular velocity of a universal joint between the two bodies i and j is defined as

$$\omega_{j,i} = \begin{bmatrix} u_i^1 & u_i^2 \end{bmatrix} \begin{bmatrix} \dot{\theta}_i^1 \\ \dot{\theta}_i^2 \end{bmatrix} \quad (5)$$

where u_i^1 and u_i^2 are two joint axes. Since the universal joint has two degrees of freedom,
θ^1 and θ^2 are the two rotational coordinates of the joint. The point connecting any two contiguous bodies is common and is shard by the two bodies.

The angular velocities of the six bodies are

$$\begin{aligned} \omega_2 &= \omega_1 \\ \omega_3 &= \omega_2 + \omega_{3,2} \\ \omega_4 &= \omega_3 + \omega_{4,3} \\ \omega_5 &= \omega_4 + \omega_{5,4} \\ \omega_6 &= \omega_5 + \omega_{6,5} \\ \omega_7 &= \omega_6 + \omega_{7,6} \end{aligned} \quad (6)$$

here ω_1 is the angular velocity of the fuselage (body 1), and $\omega_{j,i}$, is the angular velocity between bodies *i* and *j*.

By similar process, the translational velocities as

$$\begin{aligned} \dot{r}_2 &= \dot{r}_2 \\ \dot{r}_3 &= \dot{r}_2 - \boldsymbol{d_{32}}\omega_2 \\ \dot{r}_4 &= \dot{r}_2 - \boldsymbol{d_{43}}\omega_2 - \boldsymbol{d_{43}}\omega_{3,2} \\ \dot{r}_5 &= \dot{r}_2 - \boldsymbol{d_{53}}\,\omega_2 - \boldsymbol{d_{53}}\,\omega_{3,2} - \boldsymbol{d_{54}}\,\omega_{4,3} \\ \dot{r}_6 &= \dot{r}_2 - \boldsymbol{d_{63}}\omega_2 - \boldsymbol{d_{63}}\omega_{3,2} - \boldsymbol{d_{64}}\omega_{4,3} - \boldsymbol{d_{64}}\omega_{5,4} \\ \dot{r}_7 &= \dot{r}_2 - \boldsymbol{d_{73}}\omega_2 - \boldsymbol{d_{73}}\,\omega_{3,2} - \boldsymbol{d_{74}}\omega_{4,3} - \boldsymbol{d_{75}}\omega_{5,4} - \boldsymbol{d_{76}}\omega_{6,5} \\ \dot{r}_8 &= \dot{r}_2 - \boldsymbol{d_{83}}\omega_2 - \boldsymbol{d_{83}}\omega_{3,2} - \boldsymbol{d_{84}}\omega_{4,3} - \boldsymbol{d_{85}}\omega_{5,4} - \boldsymbol{d_{86}}\omega_{6,5} - \boldsymbol{d_{87}}\omega_{7,6} \end{aligned} \quad (7)$$

where the vector $d_{ij} = r_i - r_j$ and $\dot{r}_2$ represents the translation velocity of point 2.

The absolute velocities of the unit vectors are:

$$\begin{aligned} \dot{u}_3^1 &= -U_3^1\omega_2 \\ \dot{u}_3^2 &= -U_3^2\omega_2 \\ \dot{u}_4^1 &= -U_4^1(\omega_2 + \omega_{3,2}) \\ \dot{u}_4^2 &= -U_4^2(\omega_2 + \omega_{3,2}) \\ \dot{u}_5^1 &= -U_5^1(\omega_2 + \omega_{3,2} + \omega_{4,3}) \\ \dot{u}_5^2 &= -U_5^2(\omega_2 + \omega_{3,2} + \omega_{4,3}) \\ \dot{u}_6^1 &= -U_6^1(\omega_2 + \omega_{3,2} + \omega_{4,3} + \omega_{5,4}) \quad (8) \\ \dot{u}_6^2 &= -U_6^2(\omega_2 + \omega_{3,2} + \omega_{4,3} + \omega_{5,4}) \\ \dot{u}_6^1 &= -U_6^1(\omega_2 + \omega_{3,2} + \omega_{4,3} + \omega_{5,4} + \omega_{6,5}) \\ \dot{u}_7^2 &= -U_7^2(\omega_2 + \omega_{3,2} + \omega_{4,3} + \omega_{5,4} + \omega_{6,5}) \\ \dot{u}_8 &= -U_8(\omega_2 + \omega_{3,2} + \omega_{4,3} + \omega_{5,4} + \omega_{6,5} + \omega_{7,6}) \end{aligned}$$

where U_j^i is the skew-symmetric matrix associated with the vector u_j^i.

Now we have gathered all information necessary to construct the velocity transformation matrix for the wing system, therefore, the velocity transformation equation is obtained in matrix form as:

$$\begin{Bmatrix} \dot{r}_2 \\ \dot{u}_3^1 \\ \dot{r}_3 \\ \dot{u}_3^2 \\ \dot{u}_4^1 \\ \dot{r}_4 \\ \dot{u}_4^2 \\ \dot{u}_5^1 \\ \dot{r}_5 \\ \dot{u}_5^2 \\ \dot{u}_6^1 \\ \dot{r}_6 \\ \dot{u}_6^2 \\ \dot{u}_7^1 \\ \dot{r}_7 \\ \dot{u}_7^2 \\ \dot{r}_8 \\ \dot{u}_8 \end{Bmatrix} = \begin{bmatrix} \mathbf{I} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -U_3^1 & 0 & 0 & 0 & 0 & 0 \\ \mathbf{I} & -D_{32} & 0 & 0 & 0 & 0 & 0 \\ 0 & -U_3^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & -U_4^1 & -U_4^1 & 0 & 0 & 0 & 0 \\ \mathbf{I} & -D_{43} & -D_{43} & 0 & 0 & 0 & 0 \\ 0 & -U_4^2 & -U_4^2 & 0 & 0 & 0 & 0 \\ 0 & -U_5^1 & -U_5^1 & -U_5^1 & 0 & 0 & 0 \\ \mathbf{I} & -D_{53} & -D_{53} & -D_{54} & 0 & 0 & 0 \\ 0 & -U_5^2 & -U_5^2 & -U_5^2 & 0 & 0 & 0 \\ 0 & -U_6^1 & -U_6^1 & -U_6^1 & -U_6^1 & 0 & 0 \\ \mathbf{I} & -D_{63} & -D_{63} & -D_{64} & -D_{65} & 0 & 0 \\ 0 & -U_6^2 & -U_6^2 & -U_6^2 & -U_6^2 & 0 & 0 \\ 0 & -U_7^1 & -U_7^1 & -U_7^1 & -U_7^1 & -U_7^1 & 0 \\ \mathbf{I} & -D_{73} & -D_{73} & -D_{74} & -D_{75} & -D_{76} & 0 \\ 0 & -U_7^2 & -U_7^2 & -U_7^2 & -U_7^2 & -U_7^2 & 0 \\ \mathbf{I} & -D_{83} & -D_{83} & -D_{84} & -D_{85} & -D_{86} & -D_{87} \\ 0 & -U_8 & -U_8 & -U_8 & -U_8 & -U_8 & -U_8 \end{bmatrix} \begin{Bmatrix} \dot{r}_2 \\ \omega_2 \\ \omega_{3,2} \\ \omega_{4,3} \\ \omega_{5,4} \\ \omega_{6,5} \\ \omega_{7,6} \end{Bmatrix} \quad (9)$$

where
$\boldsymbol{\omega}_{3,2} = \left[u_3^1\dot{\theta}_3^1 \quad \dot{u}_3^2\dot{\theta}_3^2\right]$, $\boldsymbol{\omega}_{4,3} = \left[u_4^1\dot{\theta}_4^1 \quad \dot{u}_4^2\dot{\theta}_4^2\right]$, $\boldsymbol{\omega}_{5,4} = \left[u_5^1\dot{\theta}_5^1 \quad \dot{u}_5^2\dot{\theta}_5^2\right]$, $\boldsymbol{\omega}_{6,5} = \left[u_6^1\dot{\theta}_6^1 \quad \dot{u}_6^2\dot{\theta}_6^2\right]$, $\boldsymbol{\omega}_{7,6} = \left[u_7^1\dot{\theta}_7^1 \quad \dot{u}_7^2\dot{\theta}_7^2\right]$ and D is a skew-symmetric matrix associated with the components of the vector $\boldsymbol{d} = \left[d_1, d_2, d_3\right]^T$, which is defined for vector product operation as

$$D = \begin{bmatrix} 0 & -d_3 & d_2 \\ d_3 & 0 & -d_1 \\ -d_2 & d_1 & 0 \end{bmatrix}_{3x3}$$

and using relationships such as $d_{ik} + d_{kj} = d_{ij}$, then substitute the values of $\omega_{j,i}$ into equation (9) and comparing the equation (9) with $\dot{q} = \boldsymbol{R}\,\dot{\theta}$, the velocity transformation matrices $\boldsymbol{R}$ and $\dot{R}$ of the wing can be obtained.

RESULTS AND DISCUSSIONS

A computer program was developed to analyze the method described in this paper. To get a feel for aerodynamic model program performance, the code is used to simulate the oscillations of a rectangular wing (AR = 4) near and far of ground. Figure (6a), shows the effect of frequency ($\omega = \frac{2U_\infty k}{c}$, where k is a factor equal to 0.1, 0.3, and 0.5) on the lift, far of ground and Figure (6b), shows the same but with the ground effect (h/c = 0.25). Both figures indicated that the loads increase with increased frequency and the ground effect does magnify the amplitude of the aerodynamic loads. The results obtained are in the same behavior and trend with the ones in Ref. [7].

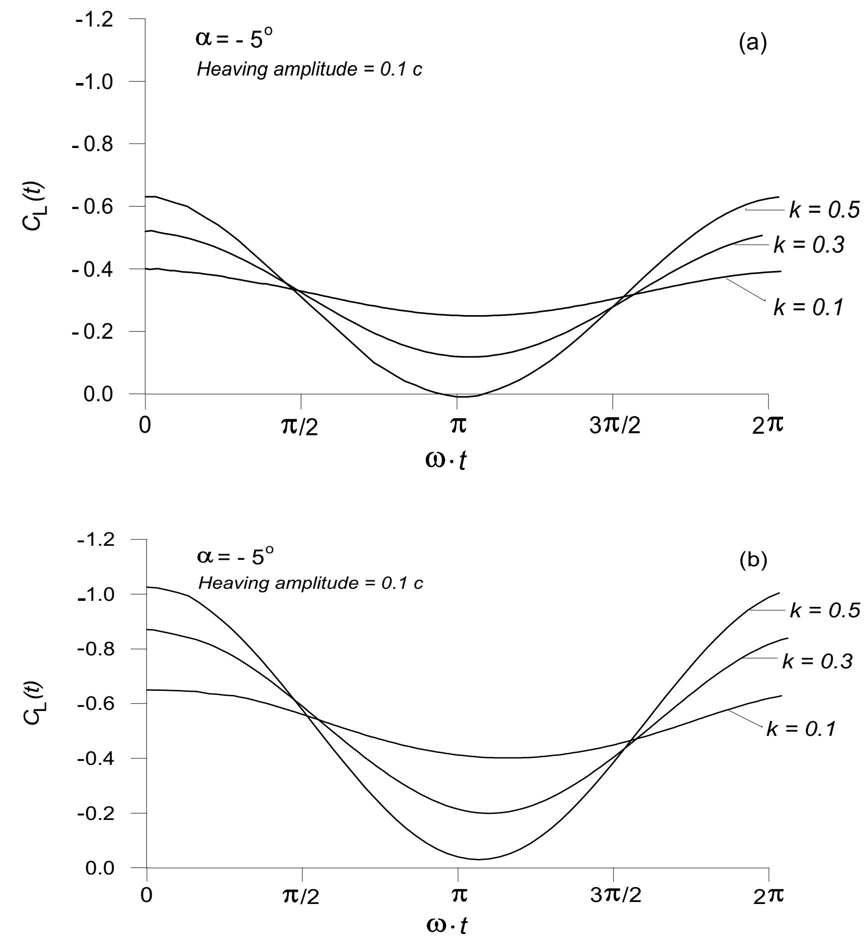


Figure 6: Effect of ground proximity on the periodic lift during heaving oscillation of rectangular wing a) Far of ground, b) near ground

To validate and asses the performance of this method, the full model of descritzing wing flying in subsonic flow is presented near and far of ground. The eigen modes for the six modes have been presented in through Figures (7-12).

As can be seen the ground has a great effect on the response of wing multibody system under consideration. Near ground the vertical displacements of the wing (bending motion) are higher than when it is far of ground due to increasing of aerodynamic loads. Thus, near ground the wing tends to vibrate more than when it is far of the ground and the ground effect tends to magnify the wing mode shapes. However, to have a feasible and better demonstration of this, the eigen value problem has to be solved, therefore, natural frequencies of the wing can be determined and hence the exact corresponding normal modes of the wing can be found and a numerical comparison can be made to check for the accuracy of this method. For flexible but not slender bodies which not going under very high rotational motion, the process of descritizing the body into small bodies instead of finite element method and the using of velocity transformation formulation, make the solution of MBS very simple and possible reducing the computer time and hence, speeded up the numerical process.

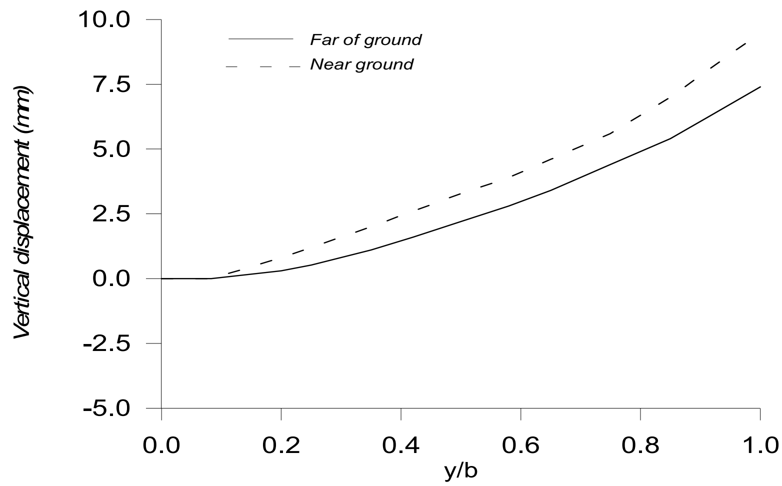


Figure 7: 1st mode shape of the discrtized wing near and far of ground

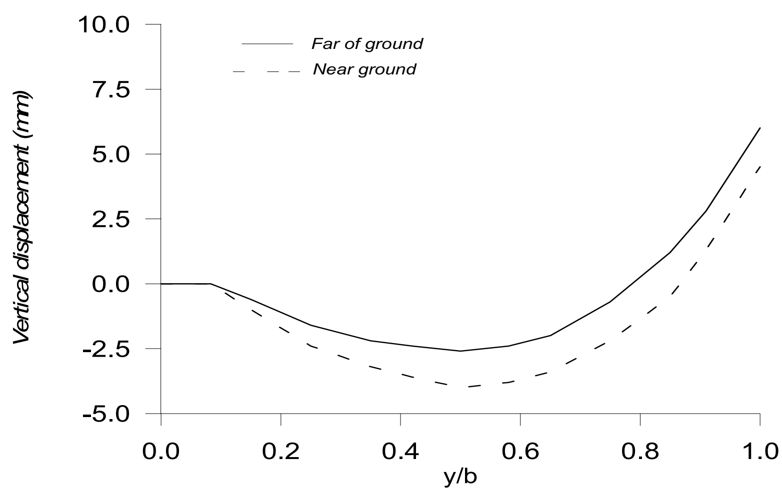


Figure 8: 2nd mode shape of the discrtized wing near and far of ground

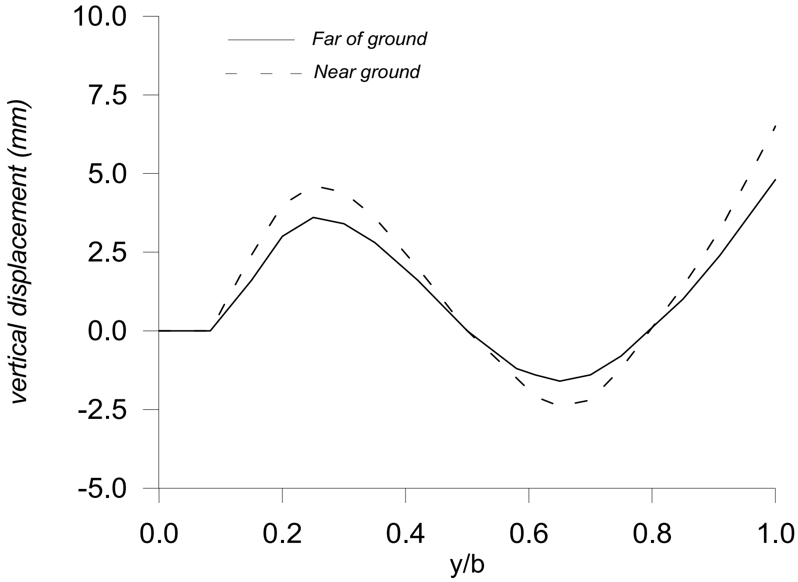


Figure 9: 3rd mode shape of the discrtized wing near and far of ground

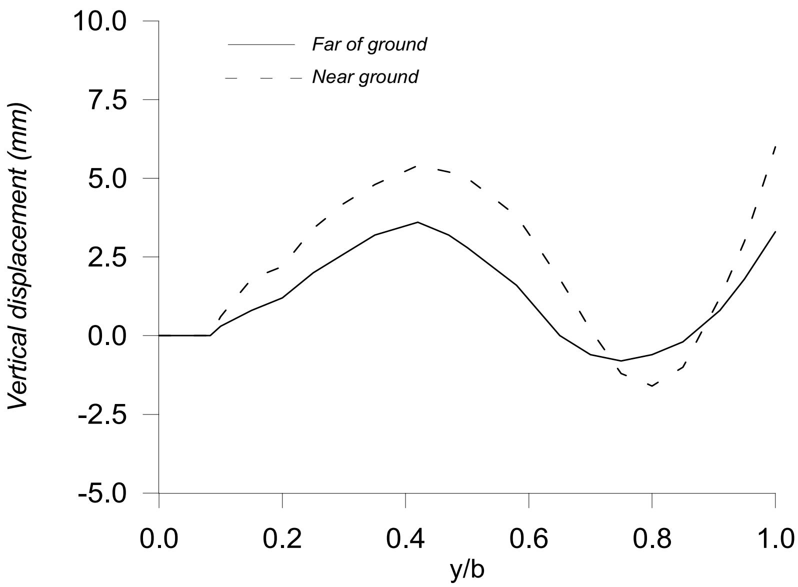


Figure 10: 4th mode shape of the discrtized wing near and far of ground

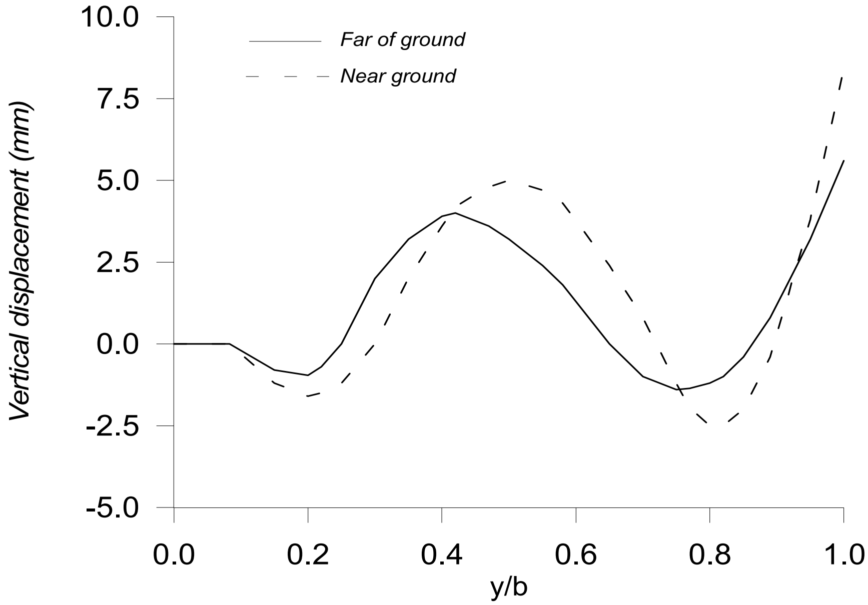


Figure 11: 5th mode shape of the discrtized wing near and far of ground

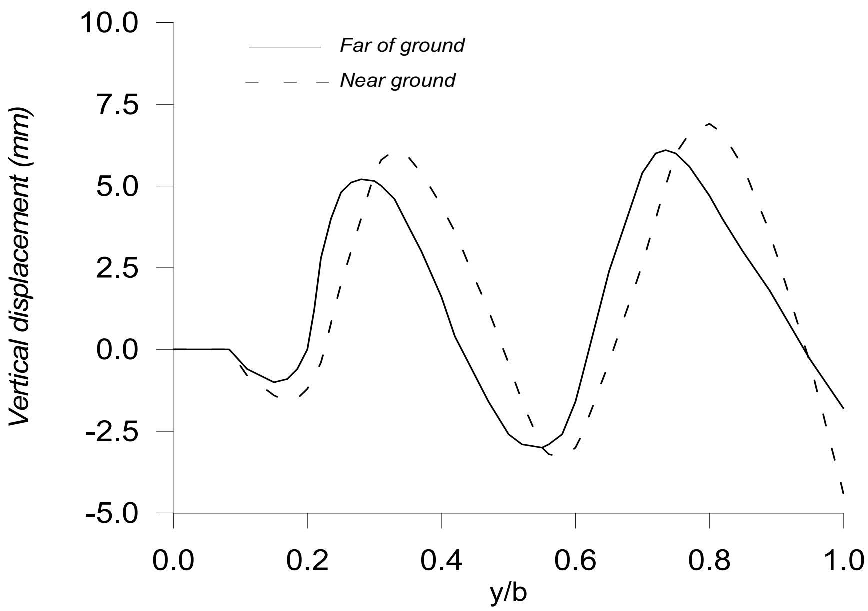


Figure 12: 6th mode shape of the discrtized wing near and far of ground

CONCLUSIONS

A mathematical model based on natural and joint coordinates for aircraft wing as multibody system is developed. The first six structural mode shapes of aircraft wing are calculated and presented. The most important conclusions based on this study are summarized;

- The method presented can be seen to be successful, yielding six associated eigen modes for the wing with root fixed end conditions and provide useful initial data, in good agreement with data that can be found in any literature for cantilever beams.
- The degree of accuracy of results obtained is dependent on the number of wing divided boxes, the more divided boxes will lead to better and more accurate results. On the other hand high computer time more sophisticated and powerful computers are needed to perform the calculators.
- The method presented here is new therefore; the results obtained can now be used for comparison with results obtained from finite element analysis.
- Finally, in general, in the analysis of complicated MBS problems, the constraint and the system equations of motion are generated and integrated using various existing computer codes. In fact the process of deriving these complicated equations for the system under consideration is time consuming and error-prone task. In turn this will have an impact on the degree of accuracy of the analysis and therefore, there is a possibility that the final results obtained might be affected.

REFERENCES

[1] Garcia de Jalon, J. and Bayo, E. "Kinematic and Dynamic Simulation of Multibody Systems, The real-Time Challenge" Springer-Verlag, 1994

[2] Huston, R.L. "Multibody Dynamics" Butterworth-Heinemann 1990

[3] Nikravesh, P.E. omputer-aided Analysis of Mechanical systems" Prentice-Hail International, 1988

[4] Shabana, A.A. "Dynamics of Multibody Systems" John Wiley and Sons, 1989.

[5] Kats, J. and Plotkin, A. "Low-speed Aerodynamic: from Wing Theory to Panel Methods" McGraw-Hill, 1991.

[6] Issa, S.M., Archewski, K.P. "Kinematics and Dynamics of Multibody System Based on Natural and Joint Coordinates Using Velocity transformations" Journal of theoretical and applied mechanices, No. 4, Vol. 36, 1998.

[7] Issa, S.M. "Dynamic of Multibody System Based on Natural and Joint Co-ordinates" Part I and II" Journal of Basic and Applied sciences, Vol. 17-Issue No. 3-2007.

[8] Laha,M.K. "A Vortex Lattice Method for Thin Wings Oscillating in Ideal Flow" Aeronautical Journal, Nov. 1993.

[9] Patel, M.H. "Aerodynamic Forces on Finite Wings in Oscillating Flow: An Experimental Study" AIAA Journal, Nov. 1978, 16.

NAMECLAURE

MBS	Mulitbody system
Θ	Joint coordinates
P, P_∞	Pressure near and far of the wing
Φ	Total velocity potential
Ω	Angular velocity of moving reference frame
V	Velocity potential

A_{ij}	Influence coefficients
RHS	Right hand side
Γ	Circulation
$U\infty$	Free stream velocity
u,v,w	Velocity components in x, y, z directions
h	Height of mean quarter chord point above ground
c	Wing chord
C	Lift coefficient