# EDUCATIONAL PROGRAM FOR STRESS ANALYSIS OF AN AIRCRAFT WING BOX STRUCTURE

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**الملخص**

تم في هذه الورقة كتابة برنامج تعليمي بلغة الماتلاب (MATLAB) ويشتغل من خلاله يقوم بحساب جميع الخواص الهيكلية مثل مركز الثقل ومركز القص وكذلك إيجاد القص الإنسيابي والإجهادات عند أي نقطة على امتداد جناح الطائرة. البرنامج يقوم بتحليل صندوق جناح الطائرة المتكون من خلية أو خليتين مدعومتين. يتكون صندوق الجناح من القشرة العلوية والسفلية والدعامات الرئيسية الأمامية والخلفية. تم مقارنة نتائج البرنامج بالنتائج المنشورة لصندوق جناح الطائرة المتكون من خلية أو خليتين مدعومتين.

تم تصنيع جناح طائرة ذات خلية واحدة مدعومة مصنوعة من مادة الألومنيوم لغرض إجراء عدة اختبارات استاتيكية تحت تأثير أحمال مختلفة لغرض المقارنة والتأكد من نتائج البرنامج. كان الجناح بطول 2400 مم، الوتر 400 مم وسمك قشرة الجناح 0.914 مم. الجناح المصنع يتكون من القشرة العلوية والسفلية سبع دعامات عرضية (أضلع) وأربع دعامات طولية والدعامتين الامامية والخلفية. البرنامج يحسب مواضع مركز الثقل ومركز القص وعزوم القصور الذاتي بالإضافة إلى إيجاد القص الانسيابي والإجهادات عند أي نقطة على طول جناح الطائرة. نتائج البرنامج الحالي متوافقة مع النتائج النظرية والعملية المتوفرة في المراجع العلمية.

**ABSTRACT**

In this paper, a computer program based on the MATLAB software has been developed. The developed program is used to compute all the structural properties such as centre of gravity, shear centre, shear flows and the stresses and any point along the wing span of a given aircraft, such parameters are not available in the MATLAB software. The generalized formulation allows performing the analysis of the wing structure for both single and double wing cells with multiple stringers. The wing box consists of upper and lower skins, stringers and front and rear spars. Case studies from the open literature are considered in this paper for the validation of the developed program for both single and double wing cells.

A wing box single cell made from aluminum materials is constructed and several static tests are carried out for the validation of structure properties. The semi-span of the wing box, chord and skin thickness are 2400, 400 and 0.914 mm respectively. The wing box consists of upper and lower skins, seven ribs, four stringers and front and rear spars.

The developed program results are in the form of locations of centre of gravity, shear centre, stiffness's, shear flows and stresses distributions at any section along the wingspan. The results of the program are in good agreement with theoretical and experimental results available in open literature.

**KEYWORDS**: Education; Program, Wing Stresses; Centre of Gravity; Shear Centre; Stiffness's; MATLAB; Shear Flow; Wing box.

## INTRODUCTION

When designing an aircraft, it's all about finding the optimal proportion of the weight of the vehicle and payload. It needs to be strong and stiff enough to withstand the exceptional circumstances in which it has to operate, [1-3].

The main sections of an aircraft, the wing, fuselage and tail, determine its external shape. Aircraft structure is usually subjected to different types of loading, air loading, inertia loading, from which three types of applied loadings are developed on the three main aircraft structures, namely shear force, bending moment and torque. Aircraft wing structure is the main lifting surface and usually carry most of the applied loads of the aircraft. As a result the wing should be designed to withstand the ultimate loads generated from the flight envelopes of the aircraft according to the airworthiness regulations.

Aircraft structure is also a very important complex structure that carry all the applied loads (static and dynamic) without structure failure. One of the important applied loads is the dynamic loads, which is affected the aeroelastic instability in the form of torsion divergence and flutter, in which the vibration characteristic in the later plays an important role through the distributions of the elastic properties such as center of gravity, shear center, bending and torsional stiffness along the span of the wing structure. The locations of the aerodynamic centre, centre of gravity and shear centre along the body and span of the aircraft and wing structure respectively are very important and may cause or lead to unstable condition and hence to poor aircraft performance. Also it will lead to structural failure as a result of any aeroelastic instabilities such as flutter or divergence speeds, in which these centre points play a very important role.

Simplifying the actual wing structure from three dimensional to one dimensional structure reduces the complexity, cost and the time of the analysis of the wing structure with an acceptable results compared with other results such as actual or experimental results in the case of vibration, aeroelastic and static (deflections) analysis, see Figure (1).

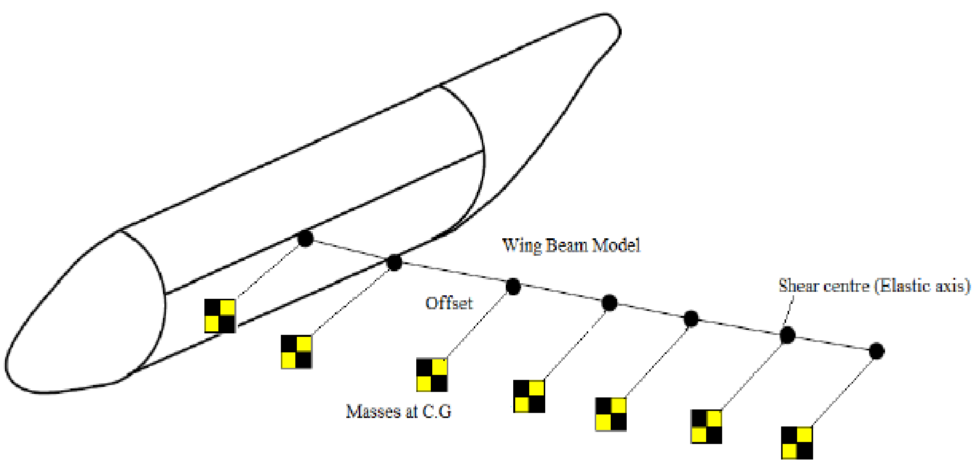


**Figure 1: Simplified wing beam with lumped masses**

In this paper, two approaches are used to obtain the structure properties of the wing box structure. The first approach is the theoretical analysis, which is done by developing a computer program within the MATLAB software to calculate the structure properties in the form of wing stiffness, centre of gravity, shear centre, shear flows and both bending stress, shear stress distribution along the span of the wing structure. The wing box is considered as single and double cells in the analysis under the three types of loads mentioned above, see Figure (2).

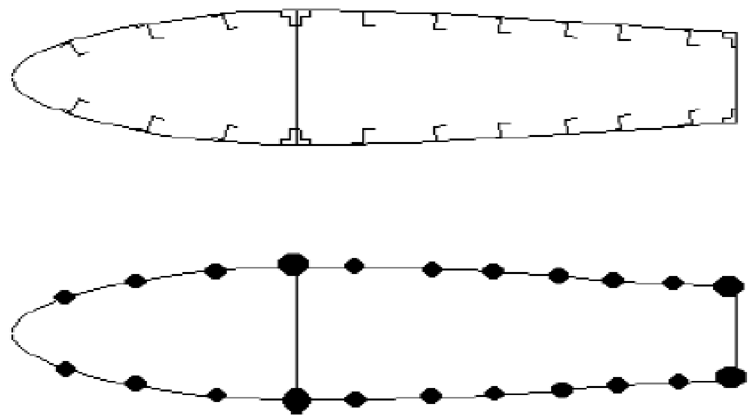

**Figure 2: Original and Idealized wing sections, [1]**

The second approach is the practical or experimental analysis. The wing box representing a single cell made from aluminum materials, close enough representing a low speed low altitude UAV aircraft wing box structure is fabricated and setup to simulate a cantilevered wing box structure. It is intended to measure the structure properties in the form vertical and angular displacements (from which both wing bending and torsional stiffness are obtained), centre of gravity, shear centre and bending moment of inertia along the span of the wing box structure. Only centre of gravity location and deflections are measured and presented here due to the limited facilities such dial gauges and other accessories. The results of both approaches are further compared and validated.

The maximum shear force, bending moment and torsion loads are assumed to be calculated separately from the aircraft loading analysis for different loading cases at any wing locations and then used as an input data to the computer program. These loads are used in the developed program to obtain the corresponding wing stresses and other properties such as centre of gravity and shear centre.

## METHOD OF ANALYSIS

### Bending Stresses

The stress at any point in the structure can be determined as a function of the applied moments at that point and the area properties of the cross-section. Equation 1 shows that the longitudinal stress is found by using the area moments of inertia, the applied moments and the x and y coordinates of the centroid of the object under stress, [1].

$$\sigma_z = \frac{M_y I_{xx} - M_x I_{xy}}{I_{xx} I_{yy} - I_{xy}^2} \, .x \; + \; \frac{M_x I_{yy} - M_y I_{xy}}{I_{xx} I_{yy} - I_{xy}^2} \, .y \qquad (1)$$

Where $M_x$ and $M_y$ are the bending moments in the structure (about "x" axis and "y" axis) and x and y are the coordinates (relative to the centroid) of the position that has to be evaluated. The procedure is to evaluate the bending stress based on [1]. It can be seen that $\sigma_z$ is strongly dependent on the distance from the section's centroid. There is a line, called the neutral axis, passing through the centroid on whose points the normal bending stresses are zero. Bending moments are vary with the location along the wing span and are maximum at the root, which is the critical section where yield or failure has to be evaluated, [2]. In an idealized wing section the bending moments are resisted by spar booms (spar flanges) with the support of the stringers only. The bending stress is calculated using the developed program at each spar boom or stringer with respect to the principle axis, u and v, see [2-3].

**Section Centroid and Moment of Inertia**

It is the point about which the weight has no moment (rotation effect). It is a three dimensional point, but it should be near enough on the centre line of a symmetrical aircraft, the height doesn't much matter, so all the important here is its fore and aft position. The section centroid and moments of inertia are computed for the actual single and double cells wing box section.

The centre of gravities in x, ($X_{cg}$) and y, ($Y_{cg}$) axes are calculated for all the stations using equations (2 and 3).

$$X_{cg} = \frac{\sum_{i=1}^{n} x_{i_{B_i}}}{\sum_{i}^{n} B_i} \quad (2)$$

$$Y_{cg} = \frac{\sum_{i=1}^{n} y_{i_{B_i}}}{\sum_{i}^{n} B_i} \quad (3)$$

Where $B_i$ Boom area of the $i^{th}$ skin-stringer.
$X_i$ distance between the reference axis and the $i^{th}$ skin-Stringer in the x-direction.
$Y_i$ distance between the reference axis and the $i^{th}$ skin-Stringer in the y-direction.
The second moment of area about the centroid are given as

$$\begin{aligned} I_{xx} &= \textstyle\sum_{i=1}^{n} B_i y_i^2 \\ I_{yy} &= \textstyle\sum_{i=1}^{n} B_i x_i^2 \\ I_{xy} &= \textstyle\sum_{i=1}^{n} B_i\, x_i y_i \end{aligned} \quad (4)$$

Where $I_{xx}$ and $I_{yy}$ and $I_{xy}$ are the area moments of inertia about the centroid, [2]. The area moment of inertia about the principle axis are calculated using the developed program in terms of $I_{uu}$ and $I_{vv}$ and $I_{uv}$ as illustrated in [3 and 6].

**Shear Centre**

Shear centre, (SC) is the point in the wing cross section at which the shear flows induced by the shear and torsional loads produce no twisting effects. The location of the shear centre is obtained by equating the total moment due to the applied shear flow and the applied moment due to the applied shear force and torsional loadsat a point along the wing cell, see equation 5.

$$\sum M_{Applied} = \sum M_{Shear\ flow} \quad (5)$$

**Shear Flow**

To compute the shear stresses acting along the plane of any cross-section of the wing, [1], thin-web shear flow theory is implemented. Shear flow is simply a shear force per unit length. The shear stress in a skin or spar panel can be computed from the shear flow by dividing by the thickness of the panel. For this theory to be applicable, the wing structure must be idealized into an arrangement of axial stress-carrying booms with finite cross sectional area and tangential shear stress carrying panels with a very small but finite thickness. The direct-stress carrying capacity of the skin sections are incorporated as additional area to stringers and spar caps, the approximation of constant shear flow

between any two adjacent stringers or spar caps can be made. Such an approximation greatly facilitates the computation of the shear flow in the wing section.
Generally shear flow is produced when the structure is subjected to the shear force and torsional loads. The calculation of the shear flow along the span of the wing is carried out using the method presented in [1 and 3-5], which based on following general equations.

$$q_s = -\left(\frac{S_x I_{xx} - S_y I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right)\int_0^s t \cdot x ds - \left(\frac{S_y I_{yy} - S_x I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right)\int_0^s t \cdot y ds + q_{s,0} \quad (6)$$

**Single Cell Wing Box**

Since the structure has been idealized, see figure 2 and the final shear flow distribution is given by the following equation,

$$q_s = -\frac{S_y}{I_{xx}}\sum_{i=1}^{n} B_i y_i + q_{s,0} \quad (7)$$

Taking moments about the points where the shear forces are applied, then this equation becomes:

$$0 = \oint P q_b \, d_s + 2 q_{s,0} A \quad (8)$$

Obtaining $q_b$ by supposing that the closed beam section is 'cut' at some convenient point thereby producing an 'open' section (see Figure (3b)). The open shear flow distribution, $q_b$ around this 'open' section is given by equation (9), [1].

$$q_b = -\frac{S_y}{I_{xx}}\sum_{i=1}^{n} B_i y_i \quad (9)$$

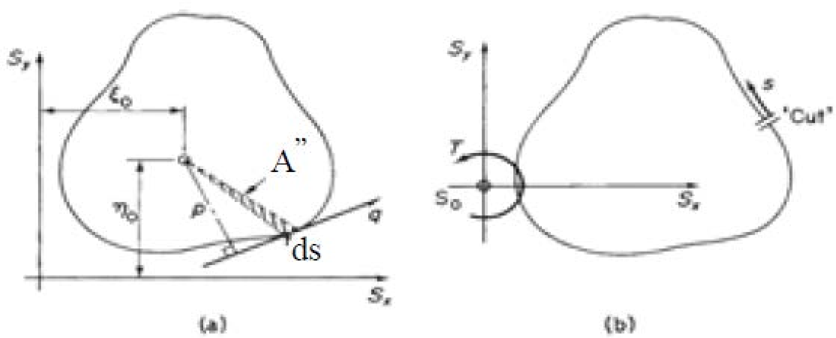


**Figure 3: (a) determination of $q_{s,o}$, (b) equivalent loading on open section beam, [1].**

The final shear flow of the single cell closed wing box is then given by

$$q_s = q_b + q_{s,0} \quad (10)$$

**Double Cell Wing Box**

The angle of twist (θ) can be calculated, which is located between the vertical and horizontal axes (x, y) and the principle axes (u, v), is presented below and in [3 and 6-7] for more details.

$$\tan 2\theta = \frac{-2Ixy}{Ixx - Iyy} \quad (11)$$

$$I_{uu} = \frac{I_{xx}+I_{yy}}{2} + \frac{I_{xx}-I_{yy}}{2}\cos 2\theta - I_{xy}\sin 2\theta \tag{12}$$

$$I_{vv} = I_{xx}\left(\frac{1-\cos 2\theta}{2}\right) + I_{xy}\sin 2\theta + I_{yy}\left(\frac{1+\cos 2\theta}{2}\right) \tag{13}$$

If a point has coordinates (x, y) from axes yy and xx then the coordinates (u, v) from principle axes vv and uu.The component of forces is calculated by using the following formulas, [3 and 6-7]:

$$S_u = S_y \sin\theta + S_x \cos\theta \tag{14}$$

$$S_v = S_y \cos\theta - S_x \sin\theta \tag{15}$$

$$q_b = \frac{S_v}{I_{uu}}\sum_{i=1}^{n} v_i\, B_i + \frac{S_u}{I_{vv}}\sum_{i=1}^{n} u_i\, B_i \tag{16}$$

$$T = 2 \times A \times q_{s,o} \tag{17}$$

Where: T: applied torque produced by the shear flow,
A: the enclosed area of the section,
$S_v$ and Su the shear force in the v-and u axis, (principle axis),
$S_y$ and $S_x$ the shear force in the y and x-axis, (centroid axis).
And finally the equation used in calculating of the angular of twist θ for the double cell wing box,

$$2G\theta = \frac{1}{A}\oint q_s \frac{ds}{t} \tag{18}$$

Where: G: shear modulus of rigidity of the materials.
Using the following equation, the moment caused by the open shear flow, $q_b$ can be obtained as

$$T = \sum_{i=1}^{n} 2\, A_i^{"} q_b \tag{19}$$

Where: $A_i^{"}$: the strip area or element area.

Equation 19 is then substituted in equation 17 to determine the balanced shear flow, $q_{s,o}$. Then the final shear flow can be calculated by the following formula:

Final shear flow, $q_s = q_b + q_{s,o}$ (20)

Finally for calculating the shear centre, the moment is taken about some point (say about the mid-point on the rear spar), and the moment is equal to double area multiplied by the shear flow, and symbolized to these strip areas with $A^{"}_i$, therefore the moment is written as:

$$T = 2\sum A^{"}\, q_s \tag{21}$$

The shear centre along the chord can be calculated at each section using equation (22):

$$SC = \frac{\sum T}{S_v} \tag{22}$$

Shear stress distributions, τ due applied shear force and applied torsion for both wing cells can be obtained as:

Shear Stress, $\tau = \frac{q_s}{t}$ (23)

Where: $q_s$: Final shear flow, t: Thickness of the web/skin.

**COMPUTER PROGRAM**

It's a well-known fact that computers have brought in a great revolution in the field of engineering education with the basic power point presentations in the class room lectures to the advanced educational software's. Educational software's have contributed to a great level in the teaching learning process; the otherwise difficult concepts for students to understand were made easy by these tools.

The students of advanced courses such as aircraft structures finds it very difficult to understand the concept of advanced mechanics of materials topics such as unsymmetric bending, shear centre and shear of open and closed symmetrical and unsymmetrical sections, for both single and double wing cells with multiple stringers and spars.Hence an educational software associated with such advanced courses will be very useful to the students and it will act as a complementary tool to traditional teaching and learning methods.

A comprehensive short term project based on available software should be given to the students to estimate final stresses on an aircraft wing will make them crystal clear about the application of these advanced topics in analysis and design of aircraft structures.

Therefore this work aims to develop a computer program within the MATLAB software for stress analysis and structure properties of wing cells with multiple stringers.The detailed input data required and the overall process of the developed program are illustrated in a simple manner through the use of flow chart shown in Figure (4 and 5).

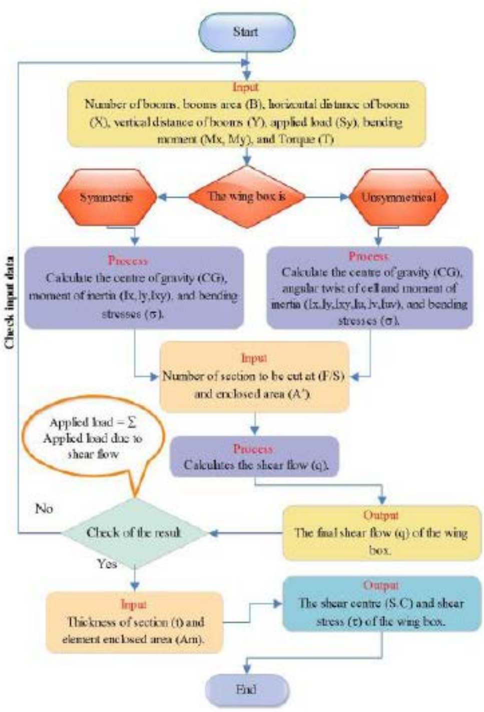


**Figure 4: Flow chart of the program for the single cell wing box**

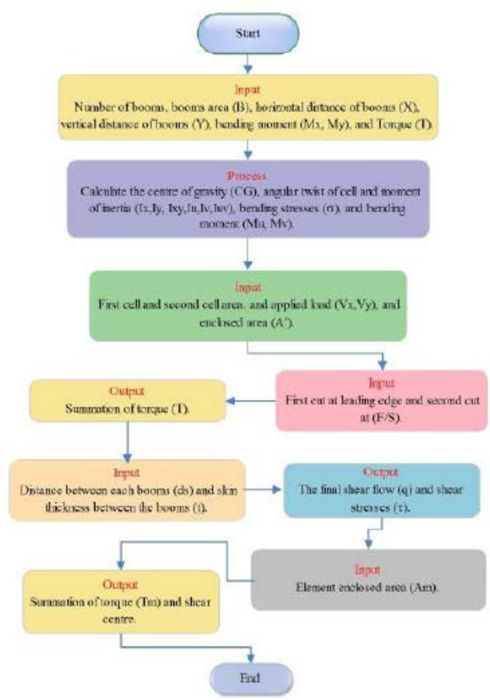


**Figure 5: Flow chart of the for the double cell wing box**

## FABRICATION OF WING BOX STRUCTURE

Wing surfaces of fixed wing aircraft generally consist of spars, ribs, skin and stringers, see Figure (6). The wing chord of small UAV aircraft is about 800 mm with the main spars located at 25% and 75% of the wing chord from the leading edge of the wing. The used airfoil section is SD7062, but because the section is cambered and due to the limitations of fabrication capabilities, the wing box is then modified to have one axis of symmetry of wing the chord, with some modification of the depth of the main wing spars. The modified dimensions of the wing box components are 400 mm chord, 110 mm and 43 mm for the front and rear spars respectively, see figure 6. The wing box properties are presented in Table (1). The fabrication of aircraft components generally involves joining of one part of the component to another.

### Fabrication Process

The first step before the fabrication of the wing box is the drawing the entire wing box structure components individually and then assembled together using the AutoCAD computer program. Based on these drawings, the following steps are summarized the fabrication process using the traditional method (available) shown in figure 7.

**Table 1: Details of the fabricated single cell wing box.**

| Component | Length | Height | Section | Thickness |
|---|---|---|---|---|
| Upper skin | 2400 | | | 20 SWG |
| Lower skin | 2400 | | | 20 SWG |
| Front Spar, F/S | 2400 | 110 | C | 20 SWG |
| F/S flange | 30 | | | 20 SWG |
| Rear Spar, R/S | 2400 | 43 | C | 20 SWG |
| R/S flange (C) | 30 | | | 20 SWG |
| Upper stringers | 2400 | 15*20*15 | Z | 20 SWG (0.914) |
| Lower stringer | 2400 | 15*20*15 | Z | 20 SWG |
| Ribs | 400 | 110 to 43 | C | 20 SWG |

Where SWG is Standard Wire Gauge.

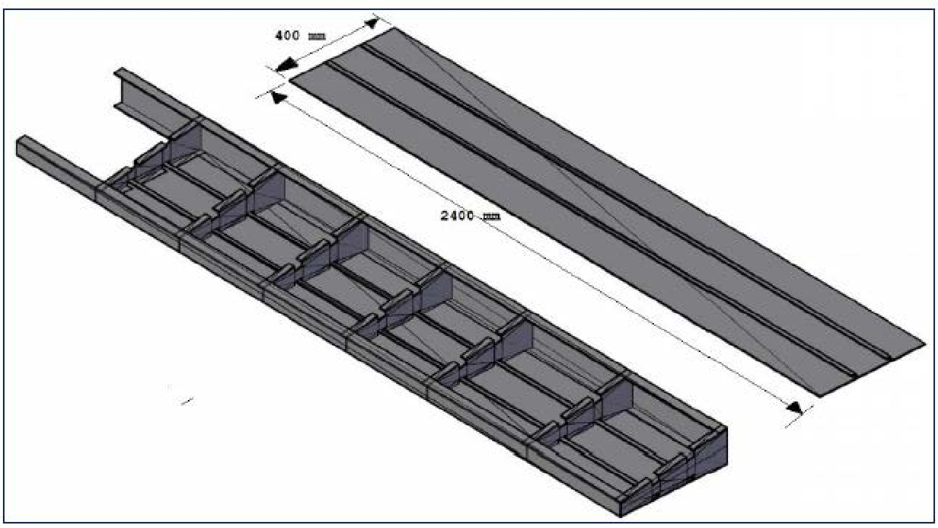


**Figure 6: General view of the wing box using AutoCAD program.**

**Figure 7: Fabrication steps of the single cell wing box.**

## CASESANALYSIS AND RESULTS

### Case Analysis 1: Fabricated Single Cell Wing Box

### Centre of Gravity

Small single cell wing box with 400 mm chord and 200 mm in length with the same section properties used in the wing box shown above is fabricated for the centre of gravity location. After many trails, the location of the centre of gravity is found and measured at 186 mm from the front spar in the chord-wise direction, at 100 mm along the length of the wing box and at zero in the vertical direction due to the symmetry of the wing box as shown in Figure (8)).

The theoretical analysis is carried out using the developed program to calculate the following the structure properties, see table 2.

1. Centre of gravity. 2. Moments of inertias. 3. Shear flow
4. Shear centre. 5. Bending and Shear stresses.

**Figure 8: Centre of gravity of the fabricated single cell wing box.**

**Table 2: Structure details of the idealized fabricated wing box.**

| Item No | Item Name | Boom Area (mm$^2$) | X (mm) | Y (mm) |
|---|---|---|---|---|
| R1 | Upper rear spar | 53 | 399.5 | 22 |
| S2 | Stringer | 68.20 | 263.967 | 32.15 |
| S3 | Stringer | 70.58 | 130.637 | 42.3 |
| F4 | Upper front spar | 62 | 0.5 | 55.25 |
| F5 | Lower front spar | 62 | 0.5 | -55.25 |
| S6 | Stringer | 70.58 | 130.637 | -42.3 |
| S7 | Stringer | 68.20 | 263.967 | -32.15 |
| R8 | Lower rear spar | 53 | 399.5 | -22 |

The structure properties, bending and shear stresses are obtained using the developed program; the results are presented in Figure (9). The difference in percentage error of the C.G. is about 2.59%, which is very acceptable at this stage; see Table (3) and Figure (10).

**Table 3: Validation of the centre of gravity from the front spar.**

| Centre of Gravity (CG), mm | | | |
|---|---|---|---|
| **Item** | **Theoretical** | **Experimental** | **Error in %** |
| CG | 190.8244 | 186.0 | 2.59 |

**Bending moment of Inertia, $I_{xx}$**

Static test is carried out to find the tip deflection of the fabricated single cell wing box under several tip concentrated loads simulating a cantilevered boundary condition, see Table (4), from which the bending moment of inertia of the fabricated single cell wing box section is then calculated.

Dial or digital gauges with stands are not available in the department to measure both the tip deflections and the angular displacements.A very simply basic method (Ruler) is used to measure the tip deflection of the wing box at different tip loads, see Figure (11). The tip loads are placed at the theoretical shear centre position to avoid any angular displacement.

However, for the angular displacement and shear centre position, the torsional stiffness of the wing box is very stiff and required a higher applied torsion loads, which is not available and also required a very strong support structure. For the above reasons, the experimental test for the shear centre is not carried out.

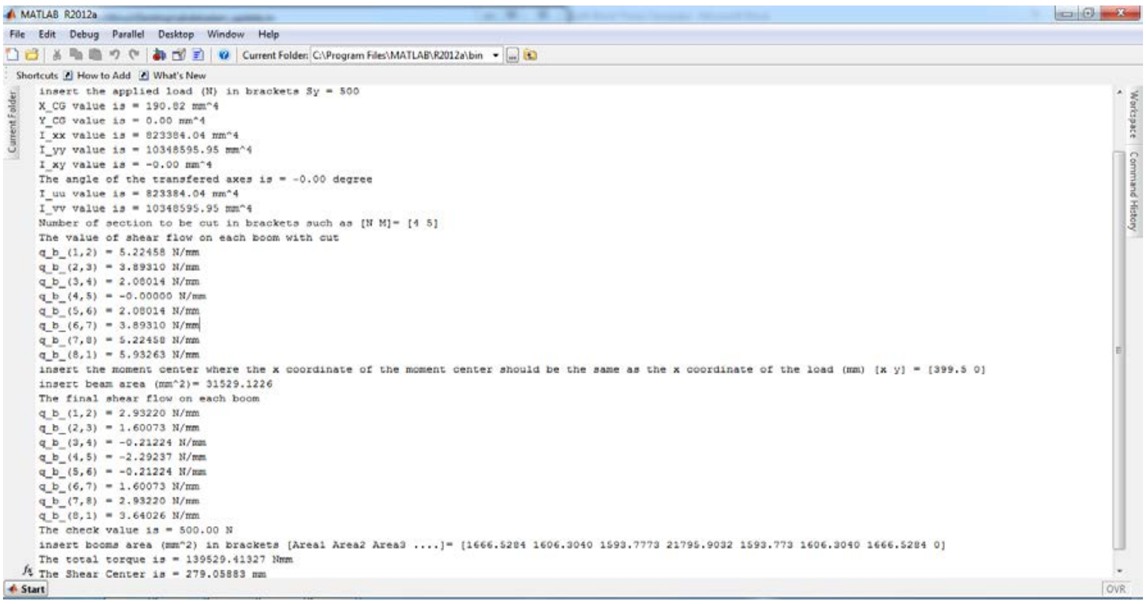


**Figure 9: Results of the developed Program for the fabricated single cell.**

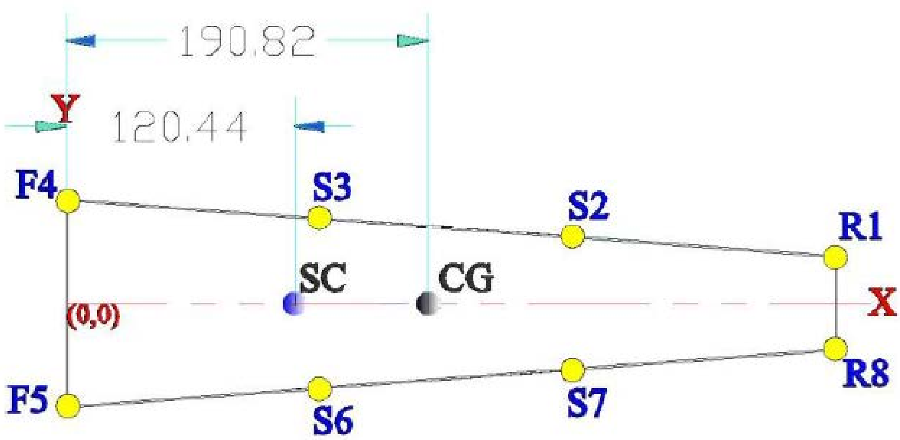


**Figure 10: Theoretical Centre of gravity and shear centre of fabricated single cell.**

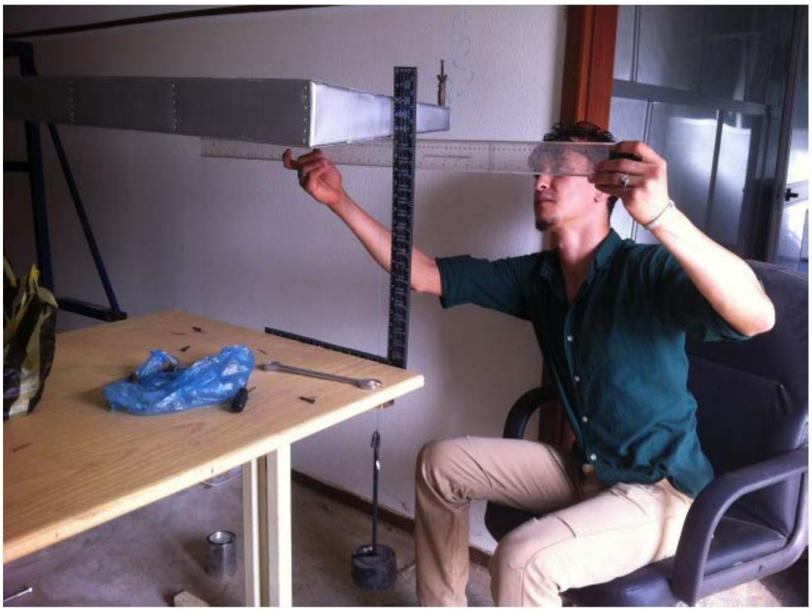

**Figure 11: Deflection tests of the fabricated single cell wing box.**

The bending moment of inertia based on the measured deflection for the cantilevered wing box under tip concentrated loads is determined using equation 24.

$$I_{xx} = \frac{PL^3}{3E\Delta_{exp}} \quad (24)$$

Where, P = Tip applied load (N).
L = Length of the wing box, (2400 mm)
E = young modules of the used materials (72000 N/mm$^2$ for Aluminum materials), [3 and 8].
$\Delta_{exp}$ = Measured tip deflection, mm

The theoretical bending moment of inertia, $I_{xx}$ is 823384.04 mm$^4$, see Figure (9)

The variations in percentage between the applied tip loadsand the measured deflection and bending moment of inertia are compared and presented in Table (4). The average values are then used for the final validation of the bending moment of inertia with the error of 8.402%, which is considered as a very good results compared with the used equipment's.

**Table 4: Theoretical and measured deflections of the fabricated wing box.**

| Load, Kg | Theoretical | Measured | Error in | Measured | Bending Moment |
|---|---|---|---|---|---|
| 0.5 | 0.38125 | 0.4 | -4.91665 | 784800 | 4.68603 |
| 1 | 0.762511 | 0.8 | -4.91651 | 784800 | 4.68603 |
| 2 | 1.525023 | 1.7 | -11.4737 | 738635 | 10.2927 |
| 3 | 2.287535 | 2.5 | -9.28793 | 753408 | 8.49859 |
| 4 | 3.050047 | 3.4 | -11.4737 | 738635 | 10.2927 |
| 5 | 3.812558 | 4.2 | -10.1622 | 747429 | 9.22479 |
| 6 | 4.575070 | 5.0 | -9.28794 | 753408 | 8.49859 |
| 7 | 5.337580 | 6.0 | -12.4105 | 732480 | 11.0403 |
| Average | 2.716447 | 3.0 | | 754199.4 | 8.402 |

**Case Analysis 2: Single Cell Wing Box, [1]**

This problem is presented in [1], page number 338 to calculate the following:

1. Centre of gravity. 2. Moments of inertias. 3. Shear flow.

For more details regarding the section properties, the reader should refer to [1].Using the equations presented above for the single cell wing box, the final shear flow distribution of the section and centre of gravity, shear centre are presented in Figure (12) and [1]. The final shear flow and structural properties of the wing box using the developed program are presented in Figure (12, 13) and Table (5).

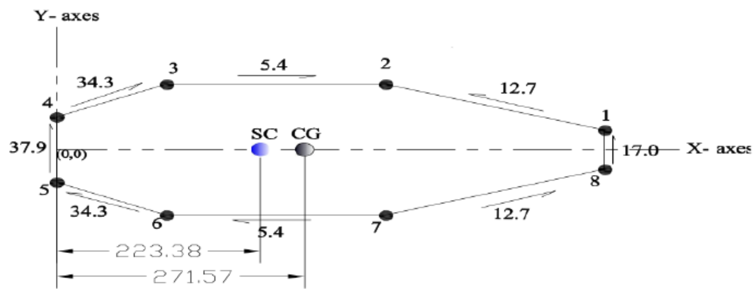


**Figure 12: Section Properties and final shear flow of the single cell wing box, [1].**

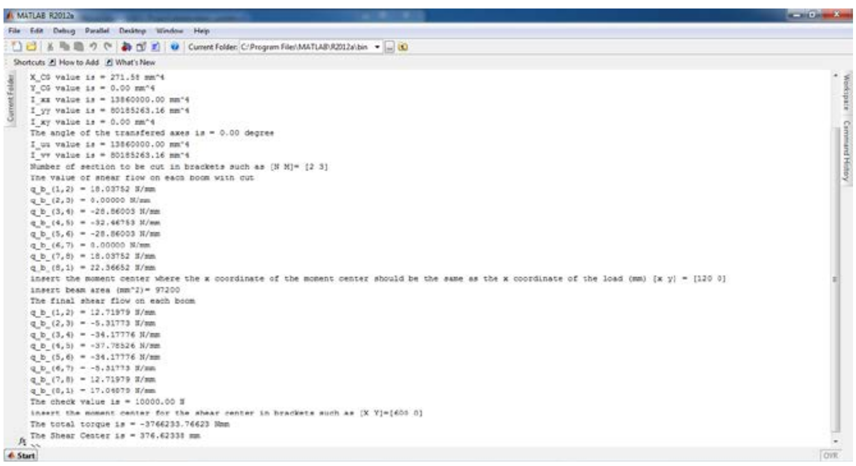


**Figure 13: Developed program results, Matlab of the single cell wing box.**

**Table 5: Developed program results of the single cell wing box.**

| Item Name | Property/No. | Final shear flow $q_s$, N/mm | |
|---|---|---|---|
| **Number of booms** | 8 | | |
| $X_{CG}$ | 271.5789 mm from (F/S) | $q_s$ (1,2) | 12.71979 |
| $Y_{CG}$ | 0 mm | $q_s$ (2,3) | -5.31773 |
| $I_{xx}$ | 13860000.00 $mm^4$ | $q_s$ (3,4) | -34.17776 |
| $I_{yy}$ | 80185263.16 $mm^4$ | $q_s$ (4,5) | -37.78526 |
| Ixy | 0 $mm^4$ | $q_s$ (5,6) | -34.17776 |
| Θ | 0.00 degree | $q_s$ (6,7) | -5.31773 |
| $I_{uu} = I_{xx}$ | 13860000.00 $mm^4$ | $q_s$ (7,8) | 12.71979 |
| $I_{vv} = I_{yy}$ | 80185263.16 $mm^4$ | $q_s$ (8,1) | 17.04879 |
| The check value | 10000.00 N | | |
| Shear center, S.C | 223.38 mm from (F/S) | | |

**Case Analysis 3: Double Cell Wing Box, [3 and 6-7]**

This problem is presented in [3, 6-7], Figure (14) below shows a typical distributed flange for unsymmetrical wing box section. The skins are stiffened by stringers and the spar caps. All the required geometrical information for stringers and spar caps are given in the following table 6. The results of the developed program are presented in Figures (15-17).

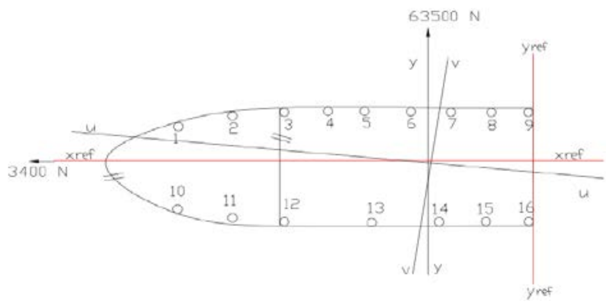


**Figure 14: Idealized unsymmetrical double cell wing box, [3, 6-7].**

**Table 6: Geometrical properties of the idealized unsymmetrical double cell, [3, 6-7].**

| Item No | Item Name | Boom Area | Y (mm) | X (mm) |
|---|---|---|---|---|
| 1 | Upper stringer | 90 | 100 | -829 |
| 2 | Upper stringer | 90 | 151 | -732 |
| 3 | Upper front spar | 245 | 175 | -621 |
| 4 | Upper stringer | 110 | 184 | -530 |
| 5 | Upper stringer | 110 | 189 | -415 |
| 6 | Upper stringer | 110 | 188 | -315 |
| 7 | Upper stringer | 110 | 182 | -215 |
| 8 | Upper stringer | 110 | 173 | -100 |
| 9 | Upper rear spar | 187 | 162 | -9 |
| 10 | Lower stringer | 110 | -83 | -831 |
| 11 | Lower stringer | 110 | -123 | -732 |
| 12 | Lower front spar | 180 | -148 | -621 |
| 13 | Lower stringer | 200 | -185 | -468 |
| 14 | Lower stringer | 200 | -203 | -310 |
| 15 | Lower stringer | 200 | -216 | -153 |
| 16 | Lower rear spar | 226 | -222 | -9 |

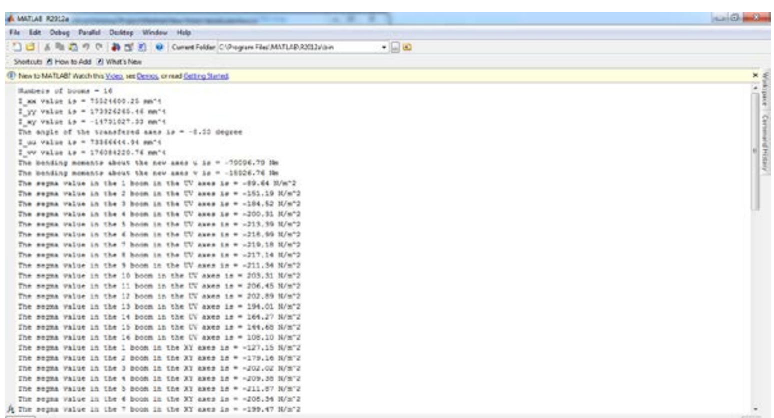


**Figure 15: Developed program results for bending stress of the double cell wing box.**

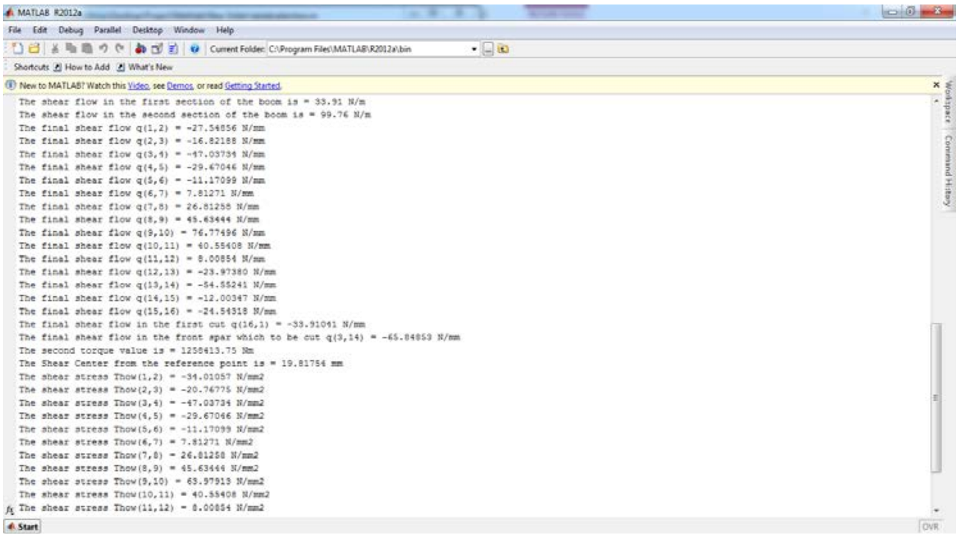


**Figure 16: Program results for shear flow and shear stress of the double cell wing box.**

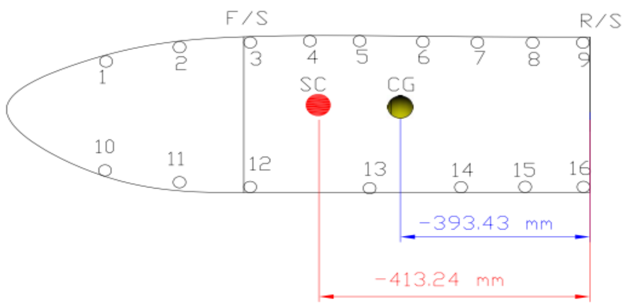


**Figure 17: Centre of gravity and shear centre for the double cell wing box.**

## VALIDATION OF THE THEORITICAL RESULTS

### Case Analysis 2: Single Cell Wing Box, [1]

Table 7 shows the comparison of results obtained by [1] and the developed computer program. The computer program results are very close (same) compared with the results obtained by [1].

**Table 7: Validation for shear Flow and centre of gravity of the single cell.**

| Shear Flow $q_s$, N/mm | | | |
|---|---|---|---|
| **Wall** | **Presented, [1]** | **Program** | **Error in %** |
| 1 2 | 12.7 | 12.71979 | 0.15 |
| 2 3 | -5.4 | -5.31773 | 1.5 |
| 3 4 | -34.3 | -34.17776 | 0.35 |
| 4 5 | -37.9 | -37.78526 | 0.30 |
| 5 6 | -34.3 | -34.17776 | 0.35 |
| 6 7 | -5.4 | -5.31773 | 1.5 |
| 7 8 | 12.7 | 12.71979 | 0.15 |
| 8 1 | 17 | 17.04879 | 0.28 |
| **Centre of Gravity (CG) mm** | | | |
| CG | 271.57 | 271.5789 | 0.003 |

**Case Analysis 3: Double Cell Wing Box, [3 and 6-7]**

Table (8, 9 and 10) show the comparison of results obtained by [3, 6-7] and the developed computer program for the double cell wing box.

**Table 8: Validation for bending stress of the double cell wing box.**

| Bending stress $\sigma_z$ (principle axis), N/mm$^2$ | | | |
|---|---|---|---|
| **Item** | **Presented, [3 and 6-7]** | **Program** | **Error in %** |
| $\sigma_{z1}$ | -90 | -89.64 | 0.4 |
| $\sigma_{z2}$ | -151 | -151.19 | 0.12 |
| $\sigma_{z3}$ | -185 | -184.52 | 0.25 |
| $\sigma_{z4}$ | -200 | -200.31 | 0.15 |
| $\sigma_{z5}$ | -214 | -213.39 | 0.28 |
| $\sigma_{z6}$ | -219 | -218.99 | 0.004 |
| $\sigma_{z7}$ | -220 | -219.18 | 0.37 |
| $\sigma_{z8}$ | -218 | -217.14 | 0.39 |
| $\sigma_{z9}$ | -212 | -211.34 | 0.31 |
| $\sigma_{z10}$ | 203 | 203.31 | 0.15 |
| $\sigma_{z11}$ | 206 | 206.45 | 0.21 |
| $\sigma_{z12}$ | 203 | 202.89 | 0.05 |
| $\sigma_{z13}$ | 194 | 194.01 | 0.005 |
| $\sigma_{z14}$ | 164 | 164.27 | 0.16 |
| $\sigma_{z15}$ | 145 | 144.68 | 0.4 |
| $\sigma_{z16}$ | 109 | 108.10 | 0.12 |

**Table 9: Validation of shear Flow, centre of gravity and shear centre.**

| Shear Flow $q_s$, N/mm | | | |
|---|---|---|---|
| **Wall** | **Presented, [3 and 6-7]** | **Program** | **Error in %** |
| 1 2 | -27.7 | -27.54856 | 0.54 |
| 2 3 | -17 | -16.82188 | 1.04 |
| 3 4 | -47.1 | -47.03734 | 0.13 |
| 4 5 | -29.8 | -29.67046 | 0.43 |
| 5 6 | -11.2 | -11.17099 | 0.25 |
| 6 7 | 7.7 | 7.81271 | 1.46 |
| 7 8 | 26.7 | 26.81258 | 0.42 |
| 8 9 | 45.6 | 45.63444 | 0.07 |
| 9 16 | 76.7 | 76.77496 | 0.09 |
| 16 15 | 40.9 | 40.55408 | 0.84 |
| 15 14 | 8.3 | 8.00854 | 3.51 |
| 14 13 | -23.7 | -23.97380 | 1.15 |
| 13 12 | -54.3 | -54.55241 | 0.46 |
| 12 11 | -11.5 | -12.00347 | 4.37 |
| 11 10 | -24.1 | -24.54318 | 0.54 |
| 10 1 | -34.1 | -33.91041 | 1.04 |
| 3 12 | -65.8 | -65.84853 | 0.13 |
| **Centre of Gravity (CG), mm** | | | |
| CG | -393.4 | -393.4305 | 0.007 |
| Shear Centre (SC), mm | | | |
| SC | 20 | 19.81754 | 0.1 |

**Table 10: Validation of shear stress of the double cell wing box.**

| | Shear Stress τ, N/mm² | | |
|---|---|---|---|
| Wall | Presented, [3 and 6-7] | Program | Error in % |
| 1 2 | -34.19 | -34.01057 | 0.52 |
| 2 3 | -20.98 | -20.76775 | 1.01 |
| 3 4 | -47.1 | -47.03734 | 0.13 |
| 4 5 | -29.8 | -29.67046 | 0.43 |
| 5 6 | -11.2 | -11.17099 | 0.25 |
| 6 7 | 7.7 | 7.81271 | 1.46 |
| 7 8 | 26.7 | 26.81258 | 0.42 |
| 8 9 | 45.6 | 45.63444 | 0.07 |
| 9 16 | 63.91 | 63.97913 | 0.09 |
| 16 15 | 40.9 | 40.55408 | 0.84 |
| 15 14 | 8.3 | 8.00854 | 3.51 |
| 14 13 | -23.7 | -23.97380 | 1.15 |
| 13 12 | -54.3 | -54.55241 | 0.46 |
| 12 11 | 14.19 | -14.81909 | 4.37 |
| 11 10 | 29.75 | -30.30022 | 0.54 |
| 10 1 | 42.09 | -41.86470 | 1.04 |
| 3 12 | 54.83 | -54.87378 | 0.13 |

## CONCLUSIONS

Structural properties, bending and shear stresses are calculated successfully using the developed computer program within the MATLAB commercial software for both single and double cells wing structure with multiple stringers made from aluminum materials. Fabrication of the wing box single cell is successfully completed, and CG location and bending moment of inertia are identified (experimentally) and obtained based on measured deflections respectively.

The results of the developed program for the three case studies are further compared and validated to the open literature and experimental results of the wing box. The error in percentage is found to be less than 3% for the centre of gravity and stresses, and less than 10% for the bending moment of inertia for the fabricated wing box.

Parametric analysis for different materials of wing, boom areas, web thickness and different loadings on the wings can be carried out using this program. Thus the program can be used for preliminary design and sizing of an aircraft wing. The present software is expected to be a useful tool to enhance the teaching and learning process of courses on aircraft structures and aircraft structural design.

## REFERENCES


[1] Megson, T.H.G, Aircraft Structures for Engineering Students, Fourth Edition, 2007,Elsevier Ltd, Oxford, UK.

[2] Sun, C. T. Mechanics of Aircraft Structures,2006, John Wiley& Sons, Inc., Hoboken.

[3] Bruhn, E.F. Analysis and design of flight vehicle structures, June 1973, Jacobs Publishing Inc, U.S.A.

[4] Niu, M.C. Airframe stress analysis and sizing, 1997, Hong Kong Conmilit Press.

[5] Experts Mind, http://www.expertsmind.com/questions/nature-of-distribution-of-bending-stress-30119745.aspx.

[6] Young J. B. Stresses in Unsymmetrical two Cell Wing Box section, Detail Stressing Course Notes, Aircraft Design Group, DES 8733-8734, 1992, Cranfield University UK.

[7] Mohamed kaheal Vibration characteristics of aircraft wing structure, MSC thesis, 2006, Mechanical engineering, Al-Mergeb University.

[8] Cutler John, Understanding Aircraft Structures, Fourth Edition, 2005, Blackwell Publishing, UK.