PREDICTION OF THE PRESSURE WAVE IN AUTOMOTIVE EXHAUST PIPES

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الملخص

إن معرفة سلوك سريان غازات الاحتراق في منظومة العادم لمحركات الاحتراق الداخلي الترددية يساعد على تصميم المخمد والأنابيب بكفاءة أكبر وبضوضاء أقل. إن فتح وقفل صمام العادم في زمن قصير جدا بينما يكون الضغط في غرفة الاحتراق كبيرا يسبب اندفاع موجات ضغطية في أنبوب العادم. ويعزي أغلب الضجيج الغير مرغوب فيه والناتج من المحرك إلى هذه الموجات الضغطية. تم اقتراح نموذج رياضي لحساب متغيرات التدفق في أنابيب العادم، والتركيز على التغير في الضغط مع الزمن عند نقاط مختلفة على طول الأنبوب. ومن أجل نمذجة التدفق في وجود انتقال الحرارة والاحتكاك يتم تبسيط المسألة باستعمال معادلات السريان وحيد الأتجاه. تم كتابة برنامج حاسوبي بلغة الفورتران لحساب سرعة وضغط الغازات في أنبوب قطره 0.04 متر وطوله 6 أمتار. يستعمل النموذج طريقة مكورمك العددية، وشروطاً تضمن الاتزان العددي. يتضح من النتائج أن النموذج قادر على التنبؤ بشكل الموجة التقريبي، الذي يختلف عن الشكل المتعارف عليه والذي يشبه الناقوس المقلوب. أوضحت النتائج كذلك أن النموذج قادر على التنبؤ بالقيم الصغرى والقصوى للضغط بخطأ لا يتجاوز 3.5% من النتائج الموجودة بالورقات والمجلات العلمية. كما تم حساب التدرج في الضغط على طول الأنبوب، ووجد أنه في المدى المقبول بناءً على المعادلة المستعملة لحساب معامل الاحتكاك.

ABSTRACT

Knowledge of the behavior of the flow of exhaust gases in the exhaust system of automotive engines help in the designing of a more efficient and more quite exhaust pipes and muffler. The exhaust valve opening and closure in a very short time while the pressure in the combustion chamber is high causes pressure waves traveling in the exhaust pipe. It is believed that these pressure waves are responsible for the unwanted noise produced by the internal combustion engines. A mathematical model is proposed for the calculation of the exhaust pipe flows, concentrating on the variation of pressure with time at different locations of the exhaust pipe. In order to simulate the complicated flow with the presence of heat transfer and friction, a system of quasi-one-dimensional model equations is employed. A FORTRAN program is developed to obtain the pressure and velocity of the exhaust gases in a 0.04m diameter, 6m long exhaust pipe. The model is using MacCormack method and a constraint for numerical stability. Results of the model showed that the shape of the pressure waves has the correct trend; however the shape does not have the well known inverted bell shape. Values of maximum pressure were predicted to within 3.5% from that found in literature. The pressure gradient along the exhaust pipe was predicted with an acceptable range depending upon the equation used for the friction factor.

KEYWORDS: Pressure Waves; Exhaust Pipes; MacCormack Method; Internal Combustion Engines.

INTRODUCTION

The pressure waves initiated during the induction and exhaust processes affect the engine operation and thus engine performance. The events of pressure waves initiated through tuned exhaust pipe intend to reduce the exhaust back pressure, and thus improve the scavenging process and combustion, and reduce engine emissions. The pressure wave propagation through the exhaust pipe can be calculated by the use of numerical methods, to solve the conservation equations of the flow through ducts, pressure wave can also be measured by actual experimental work on test engines. To make numerical simulation possible, the real exhaust pipe flow is assumed to be an unsteady, quasi one-dimensional flow. The flow is affected by area variation, wall friction, and heat loss. An accurate and fast model for simulation of exhaust pipe flows is still a research problem.

Literature dealing with the effects of gas dynamic phenomena in the exhaust manifolds of internal combustion engines dates back to the beginning of the twentieth century. Farmer [1] mentions that the performance of the four stroke engine is sensitive to the length of the exhaust pipe. He explained the basic mechanism of tuning the exhaust systems in order to size the length of the exhaust pipe. Farmer showed that the rarefaction wave generated by the reflection of the blow down pulse at the open end of the pipe returns to the exhaust port during the intake and the over lap period. Morrison [2] discussed pressure pulsations in the exhaust systems of four stroke engines and stated that engine performance is more sensitive to the pressure at the exhaust valve during the valve over lap period than the level of exhaust back pressure. The effects of manifolds for four and six cylinder engines are also discussed by Morrison [3]. By the 1950's, calculation of pressure wave phenomena in exhaust systems was attempted by Wallace [4]. Modeling techniques progressed rapidly in the 1960's, and 70's [5]. By the late of 1980's commercial software began to be used for engine performance predictions including the effect of exhaust systems.

Model Equations

To describe the flow of exhaust gases in the exhaust pipe, consider the effects of the pipe cross-sectional area variation, the heat transfer rate from the pipe, and the effect of friction at the wall; a set of equations supported by some boundary conditions must be written. Assuming quasi-one-dimensional flow of perfect gases, the equations that govern the flow are continuity, momentum, energy equations, together with the equation of state. In order for the model equations to save computing time and to give accurate predictions of the exhaust pipe flow; the governing equations are written in the normalized or conservative form as follows [6]:

$$\frac{\partial q}{\partial t}+\frac{\partial F'}{\partial x}+H=0\,. \tag{1}$$

The variables in the above equation are defined as,

$$q=\begin{bmatrix}\rho u\\ \rho uA\\ EA\end{bmatrix},\qquad F'=\begin{bmatrix}\rho uA\\ \rho u^2A+pA\\ u(EA+pA)\end{bmatrix},\qquad H=\begin{bmatrix}0\\ -p\dfrac{\partial A}{\partial x}-\rho AF_f\\ \rho A\dot{q}\end{bmatrix}. \tag{2}$$

Substitution from equation (2) in equation (1) yields the following,

$$\frac{\partial \rho u}{\partial t}+\frac{\partial \rho u A}{\partial x}=\boldsymbol{0}\,, \tag{3}$$

$$\frac{\partial \rho u A}{\partial t}+\frac{\partial [\,\rho u^{2} A+pA\,]}{\partial x}-[\,p\frac{\partial A}{\partial x}+\rho A F_{f}\,]=\boldsymbol{0}\,, \tag{4}$$

$$\frac{\partial E A}{\partial t}+\frac{\partial [\,u(\,EA+pA\,)]}{\partial x}+\rho A\dot{q}=\boldsymbol{0}\,, \tag{5}$$

Where, $E=\rho A(\,c_{v}T+\frac{u^{2}}{2}\,)\,.$ (6)

In the above equations ρ is the density in kg/m^3, u is the velocity in m/s, p is the pressure in *Pa*, E is the total energy per unit volume (J/m^3), A is the cross section area of the exhaust pipe (m^2), F_f is the wall friction force per unit mass, and $\dot{q}$ is the heat transfer rate per unit mass (W/kg).

Initial and boundary conditions

The above set of equations can become a closed form mathematical model if the initial, final and the boundary conditions are given. In the problem of evaluating the pressure along the exhaust pipe, the exhaust pressure as a function of time through the exhaust valve, which is considered the first spatial point, j=1, is given approximately from Figure (1), while the gases in rest of the pipe is considered stagnant at ambient pressure and temperature.

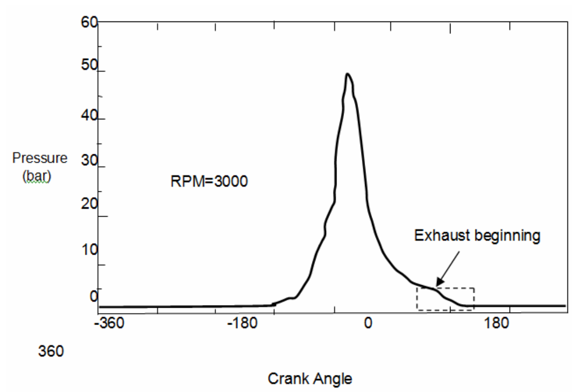


Figure 1: Pressure variation with crank angle

The solution progresses in time, and is updated during each time step until the simulation time, t_{sim}, is reached. In order to obtain a meaningful solution, boundary conditions that are physically correct and meaningful are used. The boundary conditions are expressed in terms of the primitive variables and then converted to scaled conservative variables when implemented numerically.

Numerical Scheme

To solve the above system of governing equations, The MacCormack scheme with artificial viscosity has been used. MacCormack scheme is a two-step explicit scheme and second order accurate in both space and time. Due to the presence of high gradients, the solution oscillates and it may become unstable. To have a smooth solution the artificial viscosity is added at both the predictor and corrector steps. If the solution at time t is known, then the solution at time t + Δt is found using the following two steps:

Predictor step

Continuity:

$$(\rho u)_j^{n+1} = (\rho u)_j^n - \frac{\Delta t}{\Delta x}\left[(\rho uA)_{j+1}^n - (\rho uA)_j^n\right] \tag{7}$$

Momentum:

$$\begin{aligned}(\rho uA)_j^{n+1} &= (\rho uA)_j^n - \frac{\Delta t}{\Delta x}\left[(\rho u^2 A + pA)_{j+1}^n - (\rho u^2 A + pA)_j^n\right] \\ &- \Delta t\left[(\frac{p_j + p_{j+1}}{2})(\frac{A_{j+1} - A_j}{\Delta x}) + \frac{(\rho Af)_{j+1} + (\rho Af)_j}{2}\right]^n\end{aligned} \tag{8}$$

Energy:

$$\begin{aligned}(EA)_j^{n+1} &= (EA)_j^n - \frac{\Delta t}{\Delta x}\left[(u(EA + pA))_{j+1}^n - (u(EA + pA))_j^n\right] \\ &+ \Delta t\left[\frac{(\rho A\dot{q})_{j+1} + (\rho A\dot{q})_j}{2}\right]^n\end{aligned} \tag{9}$$

State:

$$p_j^{n+1} A_j = (\rho A)_j^{n+1} R(T_j^{n+1}) \tag{10}$$

where,

$$T_j^{n+1} = \frac{1}{c_v}\left[(\frac{EA}{\rho A})_j^{n+1} - 0.5[(\frac{\rho Au}{\rho A})^2]_j^{n+1}\right] \tag{11}$$

It should be noted that all variables with the time index (n+1) are the predicted variables. These variables are replaced by the corrected variables in the next step. In this paper the first term in the last square bracket in equation (8) is zero since the exhaust pipe is of constant area.

Corrector step

Continuity:

$$(\rho u)_j^{n+1} = 0.5\left[\left[(\rho u)_j^n - (\rho u)_j^{n+1}\right] - \frac{\Delta t}{\Delta x}\left[(\rho uA)_j^n - (\rho uA)_{j-1}^n\right]\right] \tag{12}$$

Momentum:

$$(\rho uA)_j^{n+1} = \mathbf{0.5}\left[\begin{aligned}&(\rho uA)_j^n - (\rho uA)_j^{n+1} - \frac{\Delta t}{\Delta x}\left[(\rho u^2 A + pA)_j^n\right. \\ &\left.-(\rho u^2 A + pA)_{j-1}^n\right] - (p\frac{\partial A}{\partial x} - \rho Af)_j^n \Delta t\end{aligned}\right] \tag{13}$$

Energy:

$$(EA)_j^{n+1} = 0.5((EA)_j^n - (EA)_j^{n+1})$$
$$-0.5\frac{\Delta t}{\Delta x}\left[(u(EA+pA))_j^n - (u(EA+pA))_{j-1}^n\right] + 0.5(pA\dot{q})_j^n \Delta t \qquad (14)$$

State:

$$p_j^{\,n+1} = \frac{R}{2A_j c_v}\left[(EA)_j^{\,n+1} - ((\rho uA)^2/\rho A)_j^{\,n+1}\right] \qquad (15)$$

The above two steps; A and B are successively repeated until t = t_{sim}

STABILITY REQUIREMENTS

In order to obtain a stable solution, the time step must be restricted by the Courant–Friedrech-Lewy condition, CFL [7]. For inviscid flow the condition is:

$$CFL = (|u| + a)\frac{\Delta t}{\Delta x} \qquad (16)$$

The stability condition for viscous flow requires that;

$$\Delta t = \frac{CFL * \Delta x}{(a+|u|) + (2\gamma((\frac{\mu}{p_r})/(\rho R_e))}$$

$$where \qquad \mu = \frac{1.4582 * 10^{-6} T^{1.5}}{T + 110.4} \qquad (17)$$

$$and \qquad P_r = 0.9$$

Stable solution is obtained if;

$$CFL = \frac{\Delta t}{\Delta x}\left[(a+|u|) + 2\gamma(\frac{\mu / Pr}{\rho\, Re})\right] \le 1.0 \qquad (18)$$

This condition generally applied to explicit schemes for hyperbolic differential equations, physically the CFL indicates that a partial of fluid should not travel more than one spatial step size Δx in one time step Δt. In this work CFL is chosen to be 0.8

Artificial viscosity

In the high gradient region the solution oscillates and divergence may occur. To remove these oscillations and insure convergence the artificial viscosity term is added which damps out these oscillations and the solution becomes smooth and stable. In this study MacCormak-Baldwin artificial viscosity model is used; the expression below is a fourth-order numerical dissipation expression. It is equivalent to adding an extra fourth-order term to the right hand side of the modified equations for the system of difference equations, which are being solved. The fourth-order nature of the equation below can be seen in the numerators, which are products of two second-order central difference expressions for second derivatives. The MacCromack-Baldwin artificial viscosity model is defined as [8],

$$S_i^n = \frac{0.2\sqrt{(p_{i+1}^n - 2p_i^n + p_{i-1}^n)^2}}{p_i^n + 2p_i^n + p_{i-1}^n}(U_{i+1}^n - 2U_i^n + U_{i-1}^n) \qquad (19)$$

Program structure

The computer program is written in FORTRAN language, and is constructed so that the pressure results at different locations are stored in different files so that the results could be recalled and plotted. A flow chart that shows the program construction is given in Figure (2).

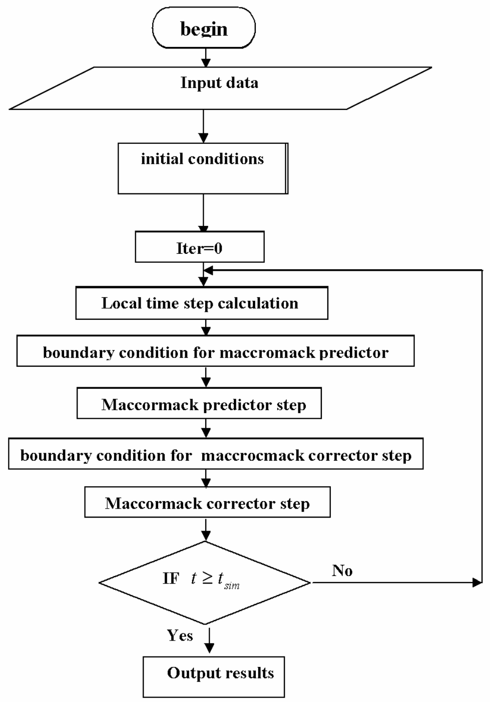


Figure2: Program flow chart

VALIDATING THE MODEL

In order to gain confidence in the numerical model of this paper, it was fed with input data given by some papers in literature and its results were compared.

Tasoi and Wang [9] developed a model to predict the maximum and minimum values of the pressure at different positions on the exhaust pipe. They compared their model results with experimental measurements, given in Table (1), and reported the percentage error of each position. The input data of reference [9], given in Table (1) was fed to the model, except for the initial pressure which is taken from the curve fit of Figure (1), and is approximated as:

$$p(t, x = \boldsymbol{0}) = \boldsymbol{506.625} - \boldsymbol{1149.4}t \quad kPa \tag{20}$$

The model results were compared with theoretical and experimental results of [9], and are given in Table (3) and in Figure (3). It can be seen that the model predicts the maximum pressures to within 5% or less. The predictions of minimum pressures were within 3% or less.

Table 1: Experimental measurements of the maximum and minimum pressures of reference [9]

Position	A	B	C	D	E	F
Max. Pressure, kPa	114.85	112.79	112.2	109.57	108.501	103.97
Min. Pressure, kPa	81.18	88.16	88.89	93.4	94.96	95.76

Table 2: Exhaust pipe geometry and the initial and boundary conditions of Reference[9]

Condition	Value
RPM	3000 rpm
Initial Pressure	Atmospheric
Initial temperature	Atmospheric
Pressure at the inlet	P(t)
Temperature at the inlet	T combustion
Length of the pipe	0.59 m
Simulation time	0.02 seconds
Ratio of specific heat	1.4
Distance of output points, m	A=0, B=0.098, C=0.196, D=0.295, E=0.39, F=0.49

Table 3: Comparison of pressure results of the model of this work with reference [9]

Position	Reference [9]				This paper			
	max	error%	min	error%	max	error%	min	error%
A	114.85	0.00%	82.42	1.53%	114.089	0.66%	82.4	-1.50%
B	113.87	0.96%	91.74	4.06%	113.088	-0.26%	89.381	-1.38%
C	111.69	-0.45%	93.8	5.52%	110.056	1.91%	91.723	-3.19%
D	110.18	0.56%	94.88	1.58%	112.23	-2.43%	91.54	1.99%
E	108.22	-0.26%	96.54	1.66%	110.47	-1.81%	92.345	2.75%
F	105.65	1.62%	98.52	2.88%	109.508	-5.33%	95.743	0.02%

It should be mentioned that the percentage error given under reference [9] in the above table (the left part) is the error when theoretical result are compared with experimental ones.

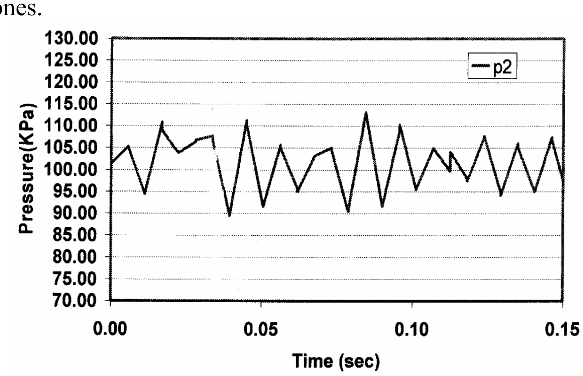


Figure 3a: Model results of pressure variation at point B of ref [9].

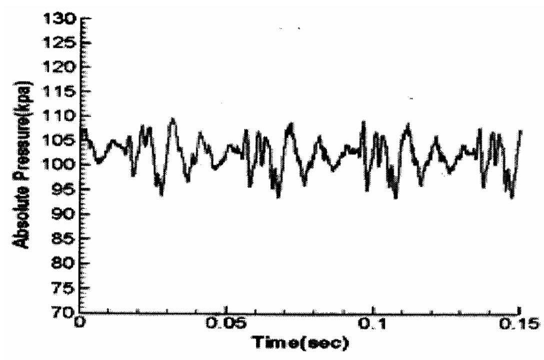


Figure 3b: Pressure variation at point B of ref [9].

THE EXHAUST PIPE GEOMETRY AND INPUT DATA

The exhaust pipe is assumed to be a straight pipe since all curvatures are smooth and with large radiuses. The pipe diameter is 0.04 m, and its length is 3.0 m. The input conditions are given in Table (4). The output pressure data is calculated at seven points at distances as shown in the table. Some necessary variables for calculations are also given in the same table.

Table 4: Input data for the current model

Condition	Value
Initial Pressure in the exhaust pipe	Atmospheric
Initial temperature	Atmospheric
Initial valve exhaust Pressure	506.625 kPa
Initial valve exhaust temperature	670 K
Diameter	0.04 m
Length of the pipe	3.0 m
Simulation time	0.02 seconds
Ratio of specific heats (air)	1.4
Output Locations	X=0, 0.52, 1.01, 1.51, 2, 2.5, 2.98 m

Discussion of Results

Even though the model is made capable of calculating many of the flow variables, the goal of this paper is to investigate only the pressure variations along the pipe, and to capture the pressure as a function of time. The pressure variation with time which is referred to as the pressure wave is calculated and plotted at different locations along the pipe, In order to have feelings about the pressure gradient; the amplitudes of the pressure waves are also plotted against the pipe length. The pressure at the open end of the pipe is assumed constant at atmospheric pressure. At the pipe entry, the pressure at zero time is set to equal the combustion chamber pressure when the exhaust valve is fully open as in equation (20). It should be noticed that the spatial grid point number is given at the right of each figure.

Figure (4) shows the variation of pressure with time at location x=0. The pressure wave amplitude was nearly 500 kPa and the time interval of the wave was about 0.005 seconds. It can be seen that the general trend of the pressure wave shape differs from the well known inverted bell shaped pressure wave.

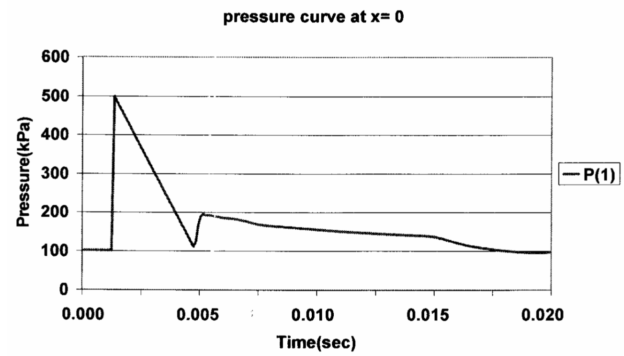


Figure 4: Pressure versus time at x=0

It should be mentioned that the shape of the pressure wave at this location is very close to the pressure variation at the valve exit when the valve is fully open. A pressure rise to nearly 200 kPa is seen to occur at 0.005 second, and then drops gradually to atmospheric pressure.

Figure (5) illustrates the pressure wave at location x=0.52 m, the pressure amplitude is shown to drop approximately to 375 kPa, and the time interval of the wave was about 0.004 seconds. Small scale fluctuations are found to exist at the peaks. The pressure wave has less amplitude value than the previous one due to the friction effect, but with similar behavior as in Figure (4).

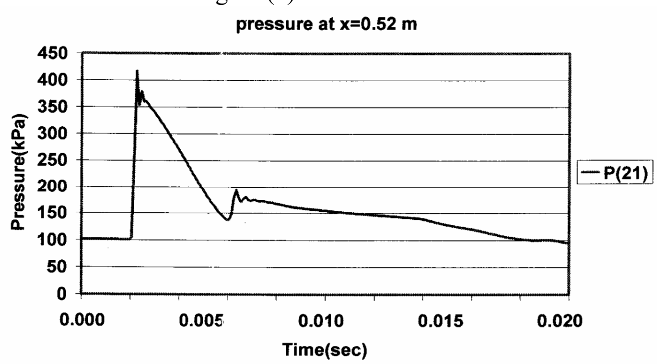


Figure 5: Pressure versus time at x=0.52 m

Figure (6) shows the pressure wave at location x=1.01m. The pressure amplitude has dropped to 300 kPa, and the time interval of the wave is about 0.0045 seconds. It can be seen that the general trend of the pressure wave shape is becoming close to the known inverted bell shaped pressure wave. No explanation can be given to the second pressure rise, or to the fluctuations at the wave peak.

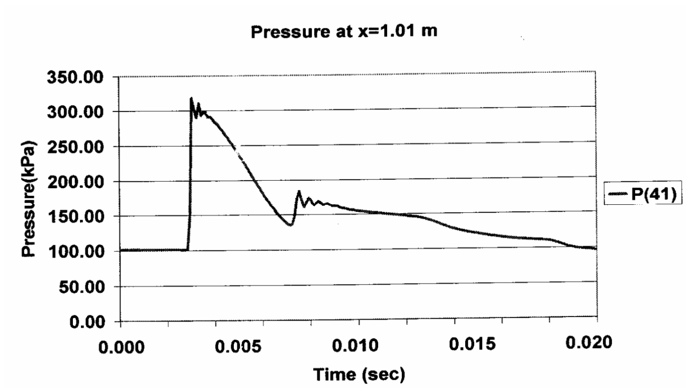


Figure 6: Pressure versus time at x=1.01m

Figure (7) depicts the pressure wave at location x=1.51m. The amplitude is seen to keep dropping to 275 kPa and the time interval of the wave is about 0.005 seconds.

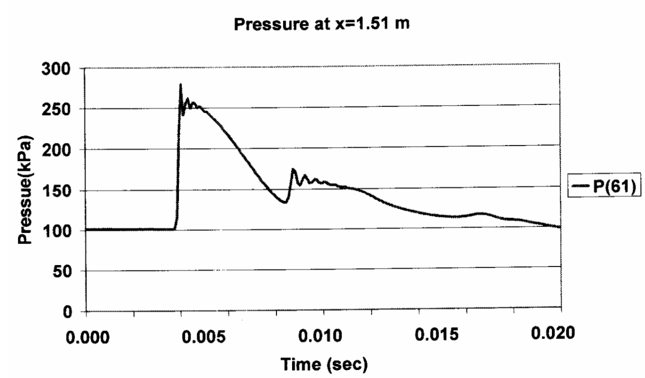


Figure 7: Pressure versus time at x=1.51 m

Figure (8) shows that the pressure wave at point x=2.0 m has an amplitude value of 230 kPa. and a time interval of about 0.006 seconds.

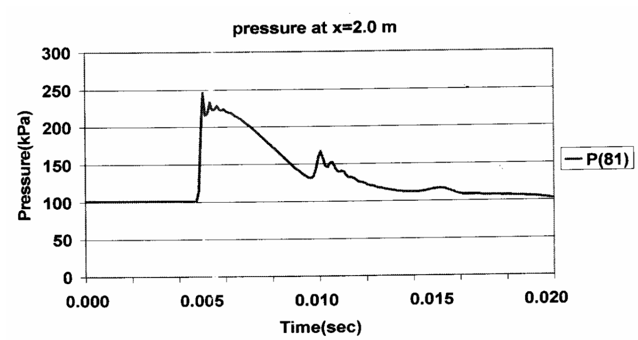


Figure 8: Pressure versus time at x=2.0 m

Figure (9) illustrates the pressure wave at location x=2.5 m. The pressure amplitude is only 210 kPa and the time interval of the wave was about 0.005 seconds. It can be seen that at this location the shape of the pressure wave is becoming more realistic.

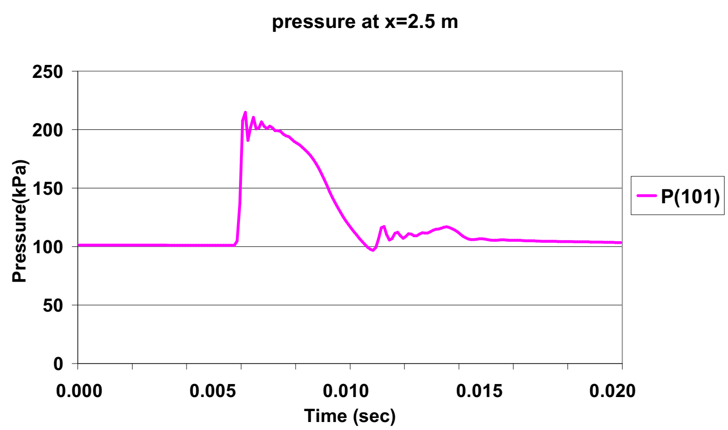
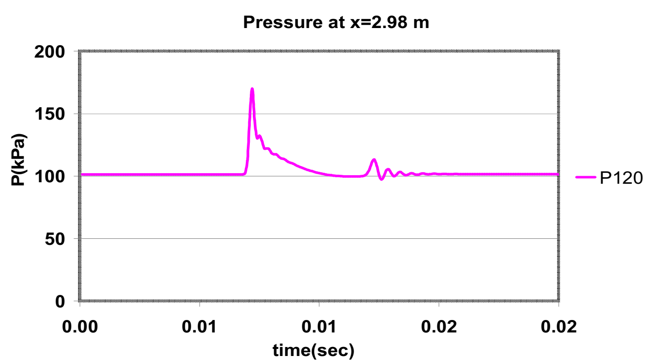


Figure 9: Pressure versus time at x=2.5 m

Finally, at the last point on the pipe, x = 2.98 m, it is decided that the pressure wave must be plotted with a much longer interval of time in order to have a general view of the wave. This general view is given in Figure (10). The pressure wave amplitude is shown to be 170 kPa approximately, and the time interval of the wave was about 0.0045 seconds. It can be seen in this figure that the pressure rise at the end of the wave is a pressure fluctuation. It is not known, however, whether this fluctuation is due to the numerical scheme, or due to physical effects.



0.005 0.01 0.015 0.02

Figure 10: Pressure versus time at x=2.98 m

The above results show that the program is capable of capturing the location, the amplitude, and the pressure rise time of the moving wave.

A final examination of the performance of the model is made on the predictions of the pressure drop along the pipe. The model is written to predict the wave pressure change with time, and no average pressure at a section is calculated. The wave maximum pressure is therefore used instead of the average pressure to indicate the pressure drop. Figure (11) shows the maximum pressure drop in the exhaust pipe.

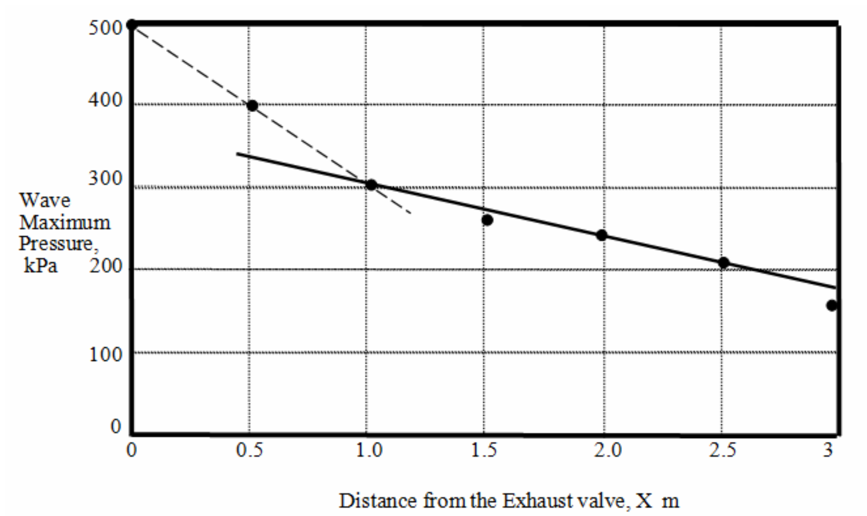


Figure 11: The maximum pressure drop along the exhaust pipe

It can be seen from Figure (11) that at the pipe entrance region, where the flow can not be considered fully developed, the pressure drop is very high, while after the entrance region length the pressure drop begins to follow a consistent trend. This trend is found to be of the same drop even when the pipe is made 7 meters long [10].

CONCLUSIONS

From the program results it can be concluded that:

- The Mathematical model could predict the maximum and minimum pressures in the combustion chamber when compared with experimental data available in literature.
- The model could predict the approximate shape of the pressure wave in locations along the exhaust pipe.
- The model could predict the amplitudes and the time intervals of the pressure waves along the pipe with accuracies of nearly 3% as in Table (3).
- The model could predict the correct trend of the pressure drop along the pipe.

- Small scale oscillations appeared on some points over the pressure waves. No clear explanation can be given. Perhaps the reason for these oscillations is the numerical scheme used.

RECOMMENDATIONS

- A more elaborate effort must be made on a more correct boundary, initial and final conditions. It is thought that the wave parameters and shape is greatly affected by these conditions. Deviation of results from reality is thought to be due to the inappropriate boundary conditions.
- A more fine revision must be made about the use of the corrector step. Many mathematical methods are used to correct the solution of the partial differential equations.
- The input pressure to the exhaust pipe is approximated from literature. It is recommended that the program handles the combustion process inside the engine cylinder to calculate the exhaust pipe input pressure.
- Nothing has been said about the second law of thermodynamic. Even though including the entropy effect complicates the model, it is recommended that a preliminary study of adding the entropy change constraint to the mathematical model must be carried out.
- Experimental facilities must be made available in the laboratories of the mechanical engineering department. Experimental I. C. Engines, replaceable exhaust systems, computer data acquisitions, pressure transducers, output data layout facilities …etc, must be made available.
- Introducing variable area requires different stability requirements. A more efficient artificial viscosity must be used.
- The program must be able to handle flows in variable area exhaust pipes. The idea is first used in this work, but it was abandoned when it is found that variable area had introduced numerical stability problem and required a plenty of time for the solution.

Nomenclature

A	Area, m^2.
a	Speed of sound, m/s.
c_v	Specific heat at constant volume,
CFL	Courant number.
E	Total energy per unit volume,
F'	Conservative variables subject to spatial derivation,
F	Wall friction force, N/kg.
f	Friction coefficient.
H	Free conservative variables,
j (subscript)	Index of space increment (grid point).
N	Number of time steps.

n (superscript)	Index of time increment (grid point).
p	Pressure, N/m^2
q	Conservative variables subject to temporal derivation,
$\dot{q}$	Heat transfer rate per unit mass, Watt/kg.
T	Temperature, K.
t	Time, seconds.
u	Velocity, m/s.
Δp	Pressure difference, Pa.
Δt	Time step.
Δx	Spatial step.
γ	Ratio of specific heats
μ	Viscosity, $N.s/m^2$
ρ	Density, kg/m^3

REFERENCES

[1] Farmer, H. O., "Exhaust System of Two Stroke Engine", Institute of Mech. Engineers, Vol.138, 1938, p 376-390

[2] Morrison,J. C., "The Problem of Exhaust Silencing and Engine Efficiency", Proc. of Auto Engineers, Vol. 27, 1932, p 614

[3] Morrison, J. C. "I. C. Exhaust System for Four and Six-Cylinder Engines, with Notes on Induction and Exhaust Gas Phenomena", Proc. of Auto Engineers, Vol 34, 1939, p 211

[4] Wallace, F. I., and Mitchell, R. W. S., "Wave Action Followinf the Sudden Release of Air Through an Engine Port System", Proc Inst. Mechanical Engineers IP, 1952, p 343

[5] Benson, R. S., and Woods, W. A., "Wave Action in The Exhaust System of Supercharged Engine Model", Int J. of Mechanical Sciences, I, 1960, p 253

[6] Anderson, J. D. Jr, 'Computational Fluid Dynamics The Basics with Applications', McGraw Hill, 1994

[7] Flitcher, C. A. J., 'Computational Techniques for Fluid Dynamics', Volume2, Springer-Verlag

[8] Alfasi, M. S., "Determination of Nozzle Performance by a Time-Dependent Numerical Technique". ICASAT, Tripoli, Libya.2007

[9] Tasi, S. J., and Wang, S. F., "An Effective Approach for Calculation of Exhaust Pipe Flow", National Cheng Kung University, Tainan, Taiwan, 70101, R.O.C.

[10] Abujnah, E. K., “Prediction of the Pressure Wave Propagation in Exhaust Pipes,” M.Sc. thesis, Mechanical Engineering Department, Tripoli University, 2010, p 56.