

# ENERGY ABSORPTION OF THIN-WALLED SQUARE TUBES SUBJECTED TO AXIAL QUASI-STATIC COMPRESSIVE LOADS

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## الملخص

تقدم هذه الدراسة تحليلاً نظرياً وعملياً لتحطم الأنابيب المعدنية رقيقة الجدار وذات المقطع المربع تحت الأحمال المحورية الضغطية الشبه ساكنة وذلك لمعرفة مقدرتها على امتصاص الطاقة الناتجة من عملية التحطيم وتحديد أساليب التحطم والأحمال المسلطة. ولغرض إجراء التجارب العملية تم إعداد العديد من عينات الأنابيب المعدنية مساحة مقطعها 100 ملليمتر ذات سماكات تبدأ من 0.5 ملليمتر إلى 2.5 ملليمتر وأطوال مختلفة. أدت التجارب إلى الحصول على مخططات الحمل مع الإزاحة لكل عينة متحطمة وكذلك حددت أساليب التحطم بنوعين. من خلال هذه الدراسة أستنتج أن الطاقة الناتجة من تحطم الأنابيب تمتص من خلال المفاصل الثابتة والمتحركة المتكونة بجدار الأنبوب أثناء التحطم. كذلك وجد أن الحمل المتوسط والحمل الأقصى والطاقة الممتصة وتناسب طردياً مع سماكة الأنبوب.

## ABSTRACT

This paper investigates the collapse behavior of thin-walled metal square tubes under axial quasi-static compressive loads. Several specimens of tubes were manufactured from carbon steel sheets with different thicknesses and lengths. The cross-sectional area of a tube is constant (100 cm<sup>2</sup>). An analytical in-extensional collapse mode was used to predict the mean load carrying capacity and the energy absorption. Load-displacement curves were obtained experimentally and two types of collapsed modes were observed. It was noted that the energy of the crushing was absorbed by fixed, included and traveled plastic hinges. Mean load carrying capacity and energy absorption was found to increase with the increase of tube thickness.

**KEYWORDS:** Collapse Mode; Energy Absorption; Thin-Walled Square Tubes; Plastic Hinges; Specific Energy

## INTRODUCTION

Energy absorption in field of automotive structures is very important when one considers the effect of their collisions on human life and safety in general. The design and development of energy absorbing devices have received much attention from mechanical engineers. The development and detailed design in controlled manner or at a predetermined rate have become increasingly more important to engineers. The advances in technology have led to higher speeds of different vehicles. Tubular structure is among energy absorption devices which can be used in many applications such as cars, ships, airplanes, etc. The advantage of the tubular structure is that it has simple geometry, low cost and high energy absorption capability. The crushing behavior of thin metal tubes was investigated by many of investigators. For example, Johnson and Soden [1] studied the crumpling behavior of thin metal tubes. Johnson and Reid [2] introduced a proximate analysis of the collapse mode and expression of the mean load

carrying capacity of cylindrical shell under axial load. There are different ideas and methods, which have been used to investigate the energy absorption of a tubular structure under different loading conditions. Rawlings and Shetland [3] reviewed a number of energy absorption devices and suggested that progressively crushing tubes under axial load are among the more effective ones. Several parameters that affect the behavior of energy absorption were also briefly discussed. Saied and Shuaeib [4] studied the energy absorbed of circular cross section metal tubes under static load. They noted that the energy absorption of crushed tubes increases with the increase of (L/D) ratio. Box tubes with either rectangular or square cross-section were also investigated to study their collapse behavior under static and dynamic loading condition. Meng and Al-Hassani [5], Wierzbicki and Abramowitz [6] and Abramowicz and Jones [7] carried out axial crushing tests on thin-walled rectangular and square cross-section tubes to study their collapse modes and evaluate their mean crushing loads. Mamalis et al [8] carried out axial static compressive tests on thin-walled octagonal steel tubes which probably were considered as in-between shape exhibiting same behavior of both cylindrical and square tubes. Yang [9] studied the dynamic progressive buckling of mild steel and aluminum square tubes. He found that most of the aluminum tubes suffer symmetric crushing and some of the mild steel tubes suffer extensional crushing. Meon et al [10] studied the effect of variation of the tube length and the crosshead velocity on the amount of energy absorption of aluminum tubes (Al 6061). They observed that the presence of friction at the die-tube interface significantly influences the energy absorption of aluminum tubes as well as its deformation.

In order to predict the mean load carrying capacity and the energy absorption of the crushed tubes, many of models have been proposed. For example Wierzbicki and Huang, [11] presented different collapsible modes to explain the manner in which thin-walled tubes subjected to axial load, crumple into series of flat, triangular and trapezoidal sheets. The mechanisms assumed basic in extensibility of the tube mid-surface, all deformation being accomplished by bending and mainly concentrated at hinge lines. Aljawi [12] studied experimentally the crushing behavior of square mild steel tubes by using the nonlinear finite element code ABAQUS. He demonstrated that good agreement was found between the FEM force histories of tubes with those obtained by experimental results for quasi-static loading. Javad et al [13] investigated energy absorption of thin-walled tubes with different geometric dimensions using finite element simulation and they found that the ellipse cross section has more energy absorption than the others. Also, they demonstrated that the amount of energy absorption increased was with increase of the thickness for smaller section tubes.

## MATERIAL AND EXPERIMENTS

### Material

The material used for this study was carbon steel sheets type ASTM code A242 with chemical composition shown in Table (1). The yield stress and other mechanical properties of the steel sheets were determined experimentally using tensile test on coupons prepared from the same sheet according to the S-A370 method [14] as shown in Table (2).

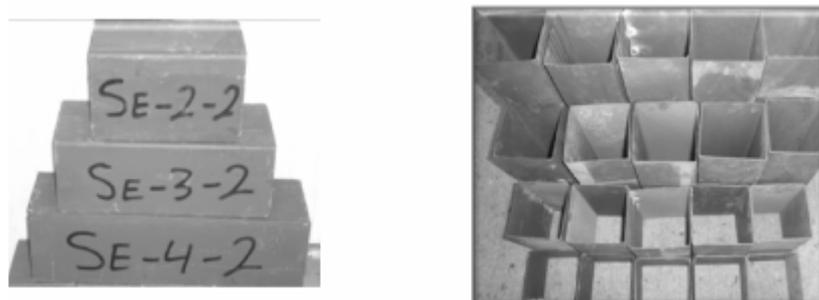
**Table 1: Chemical composition of the mild steel sheet [14]**

Alloy elements	C	Mn	Si	Cu
Wt%	0.15	0.6	0.3	0.02

**Table 2: Measured mechanical properties of the steel sheets [14]**

Yield strength (MPa)	Tensile strength (MPa)	Elongation %	Young's modulus (MPa)
300	450	20	205GPa

Specimens of 100 mm square cross-section with different thickness and lengths were manufactured from steel sheets using a hard steel mandrel, which has same dimensions of the specimens, that to be insured all of specimens have same cross-section dimensions. The edge of a bent sheet of the tube was joined by arc welding (in case of thicker sheets) or by acetylene welding (in case of thinner sheets). Each specimen was identified by numbers and letters as shown in Figure (1), where the two letters (S E) denote a square empty tube, first number indicates length to width ratio (L/D) and the last number indicates the thickness of the tubes. Table (3) illustrates the matrix of the length and thickness of the specimens. All the tubes were cleaned and painted with rust paint.



**Figure 1: The specimens with different lengths and thickness**

**Table 3: Numbering of the specimens**

L (mm)	100	200	300	400
t (mm)				
0.5	SE-1-0.5	SE-2-0.5	SE-3-0.5	SE-4-0.5
1	SE-1-1	SE-2-1	SE-3-1	SE-4-1
1.5	SE-1-1.5	SE-2-1.5	SE-3-1.5	SE-4-1.5
2	SE-1-2	SE-2-2	SE-3-2	SE-4-2
2.5	SE-1-2.5	SE-2-2.5	SE-3-2.5	SE-4-2.5

### Test Method

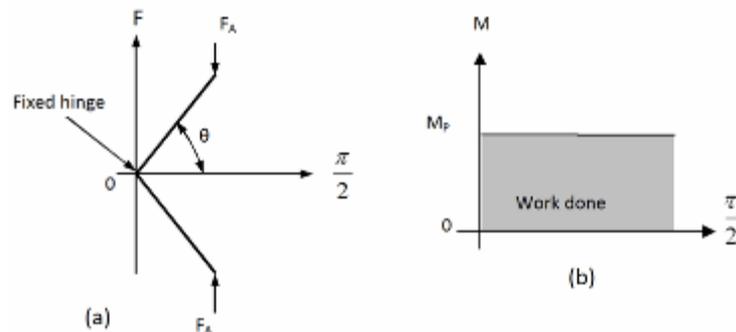
The crushing test was started by setting the servo-hydraulic universal testing machine type (Ibertest) that was equipped with appropriate load cell. The specimen was placed between the flat plates so that the ends of the tube are at right angle with plates of the machine. The load was applied gradually at speed of (10 mm/min) of cross-head until the specimen deforms completely or until it is flattened. The test machine was connected to a computer through data acquisition connections in order to obtain a load-displacement curve for each a deformed tube. A digital camera was used to take photo of the deformed tube during the crushing process. All the tests were carried out in the Materials Laboratory of the Industry Research Center at Tripoli- Libya.

### THEORETICAL ANALYSIS

#### Concept of Plastic Hinges

The concept of the work done in the plastic hinges for a collapsed tube can be modeled by a plate with unit width subjected to an axial compressive force; see Figure

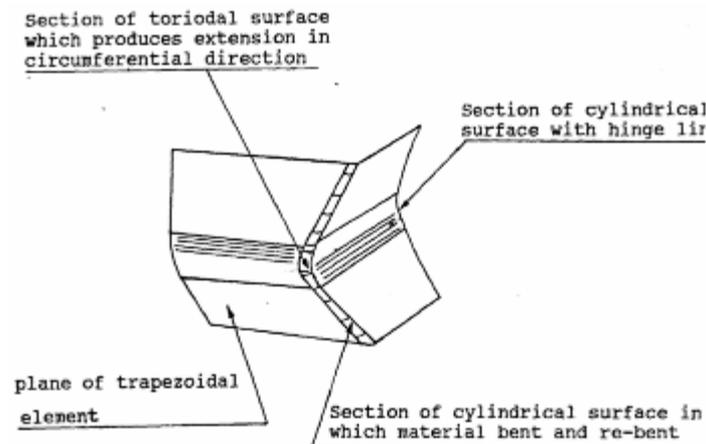
(2a). At any value of subtended angle ( $\theta$ ), the work done can be indicated by the area under the curve as shown in Figure (2b). This energy is dissipated through formation of the hinge. This concept has been used to evaluate the whole energy absorption, mean load carrying capacity of collapsed tube under static and dynamic loading conditions [3-7].



**Figures 2: (a) A metal plate under an axial compressive load, (b) bending Moment angle curve for the plastic deformation of the plate**

### Basic Fld Mechanism for Square Tube

Wierzbicki and Abramowicz [6] developed the basic fold mechanism of square tube subjected to axial compressive load as shown in Figure (3). They assumed that the square tube collapses by inextensional and extensional collapse modes. All deformations are due to local bending. The tube ends are free to deform and the idealized material of the tube is assumed to be homogeneous and rigid perfectly plastic.



**Figure 3: Construction of the basic fold mechanism of a quadrant square tube under axial compressive load [6]**

The basic fold mechanism consists of:

- Four plane trapezoidal elements moving as rigid bodies. No energy is dissipated in this case.
- A section of a toroidal element which produces extension in circumferential directions. The associated rate of energy dissipated is  $\dot{E}_1$ .
- Two sections of cylindrical surface with moving hinge lines. The deformation is inextensional and associated rate of energy is  $\dot{E}_2$ .
- Two sections of a cylindrical surface in which material is bent and re-bent again by inclined hinge. The associated rate of energy is  $\dot{E}_3$ .

The summation of the above three rates of energy dissipation ( $\dot{E}_1$ ,  $\dot{E}_2$  and  $\dot{E}_3$ ) must be equal to the rate of external work done in compressing the basic fold mechanism as:

$$\dot{E}_{\text{ext}} = P\dot{\delta} = \dot{E}_1 + \dot{E}_2 + \dot{E}_3 \quad (1)$$

Where  $P$  is the compressive load and  $\delta$  is the crushing distance.

### Mean Load Carrying Capacity

Wierzbicki and Abramowicz [6] evaluated the general form for the mean load of the basic fold mechanism by using the above expression as:

$$\frac{P_m}{M_0} = A_1 \frac{b}{h} + A_2 \frac{c}{H} + A_3 \frac{H}{b} \quad (2)$$

where;  $M_0$  is the plastic bending moment per unit width,  $A_1$ ,  $A_2$  and  $A_3$  are parameters depend on the type of the structure in question.  $C$  and  $h$  are the width and thickness of the element respectively.  $H$  and  $b$  are the instantaneous values of the half fold length and radius of toroidal surface respectively. The mean values of  $H$  and  $b$  were found to be constant during the deformation process. These values can be found by minimizing the mean load ( $P_m$ ) in equation (2) with respect to  $H$  and  $b$ . More details about the values of  $E_1$ ,  $E_2$  and  $E_3$  are in Ref [6,].

The mean load ( $P_m$ ) carrying capacity for the symmetrical collapse mode of a square tube made of rigid plastic material takes the form [6, 12] as:

$$P_m = 9.56 \sigma_0 t^{3/2} D^{1/2} \quad (3)$$

while for asymmetrical collapse mode takes form[6,12]:

$$P_m = 13.06 \sigma_0 t^{3/2} D^{1/2} \quad (4)$$

where,  $\sigma_0$  is the yield stress,  $D$  is the width of the square tube and  $t$  is the tube thickness.

### Specific Energy Absorption

The specific energy absorption ( $E_{\text{specific}}$ ) is the most important factor in the design of the parts that are needed to reduce their weight, such as cars, airplanes and motorcycles, etc. Specific energy is the energy absorbed per unit mass of the specimen. Therefore it has been expressed in term of instability of the tube. It can be calculated from the equation:

$$E_{\text{specific}} = \frac{E_{\text{total}}}{\text{mass}} \quad (5)$$

## RESULTS AND DISCUSSION

Tables (4a), (4b), (4c) and (4d) summarize the results of the experimental crushing tests and theoretical calculations for the tubes with ( $L/D$ ) ratios as indicated. It can be observed that the maximum and mean load carrying capacities, energy absorption and specific energy are increasing considerably with the increase of thickness of the tube. Reasonable agreement between theoretical and experimental values of mean load can be observed in most of the regular modes whereas the experimental values of mean load of the irregular modes did not fit with theory due to the random deformation and instability of the tubes.

**Table 4.a ( L/D = 1)**

Thickness (mm)	0.5	1	1.5	2	2.5
Mass (g)	175	314	471	628	785
Compression (%)	80	80	80	80	80
Max. load (kN)	15.3	59.5	116.5	161.2	315
Mean load (Exp.) (kN)	8.623	16.76	30.26	41.55	75.23
Mean load (theo.) (kN)	6.009	16.7	30.096	42.252	73.71
Energy absorption (J)	522	322	2500	2289	6518
collapse mode	Irregular	Irregular	Regular	Regular	Irregular
Specific Energy (kJ/kg)	2.98	1.07	5.99	3.63	8.303

**Table 4.b ( L/D =2)**

Thickness (mm)	0.5	1	1.5	2	2.5
Mass (g)	350	628	942	1256	1620
Compression (%)	75	75	75	75	75
Max. load (kN)	14.6	48.4	117	178	296.7
Mean load( exp.) (kN)	8.85	13.25	35.68	44.87	75.25
Mean load ( theo.) (kN)	5.697	13.3	35.568	42.252	73.71
Energy absorption (J)	268	1044	1879	2592	6150
collapse mode	Irregular	Regular	Regular	Regular	Regular
Specific Energy (kJ/kg)	0.76	1.662	1.99	2.06	3.79

**Table 4.c ( L/D = 3)**

Thickness (mm)	0.5	1	1.5	2	2.5
Mass (g)	525	942	1413	1844	2355
Compression (%)	60	60	60	55	55
Max. load (kN)	14.4	49.7	109.4	152.7	276.5
Mean load (kN)	8.31	21.25	32.6	40.571	70.4
Mean load (theory) (kN)	5.697	18.09	32.8	42.252	70.5
Energy absorption (J)	366	1203	2527	3326	6060
collapse mode	Irregular	Irregular	Regular	Irregular	Regular
Specific Energy (kJ/kg)	0.64	1.277	1.788	1.803	2.57

**Table 4.d (L/D = 4)**

Thickness (mm)	0.5	1	1.5	2	2.5
Mass (g)	700	1256	1884	2512	3140
Compression (%)	70	70	70	65	65
Max. load (kN)	14.39	44.86	141.2	160.4	289.8
Mean load (kN)	9.757	25.256	45.8	68.8	68.5
Mean load (theory) (kN)	5.67	17.898	35.55	57.447	67.31
Energy absorption (J)	1174.17	1479.36	2908.06	4696.69	6812.53
collapse mode	Irregular	Irregular	Irregular	Irregular	Regular
Specific Energy (kJ/kg)	1.677	1.177	1.5	1.869	2.169

### Load- Displacement Curves

The area under the load-displacement curve of a deformed tube represents the effective energy absorption of the collapsed tube and the characteristic of the curve records the behavior of the tube during deformation. Figure (4) shows the load-displacement curve for tubes with different (L/D) ratios and wall thicknesses.

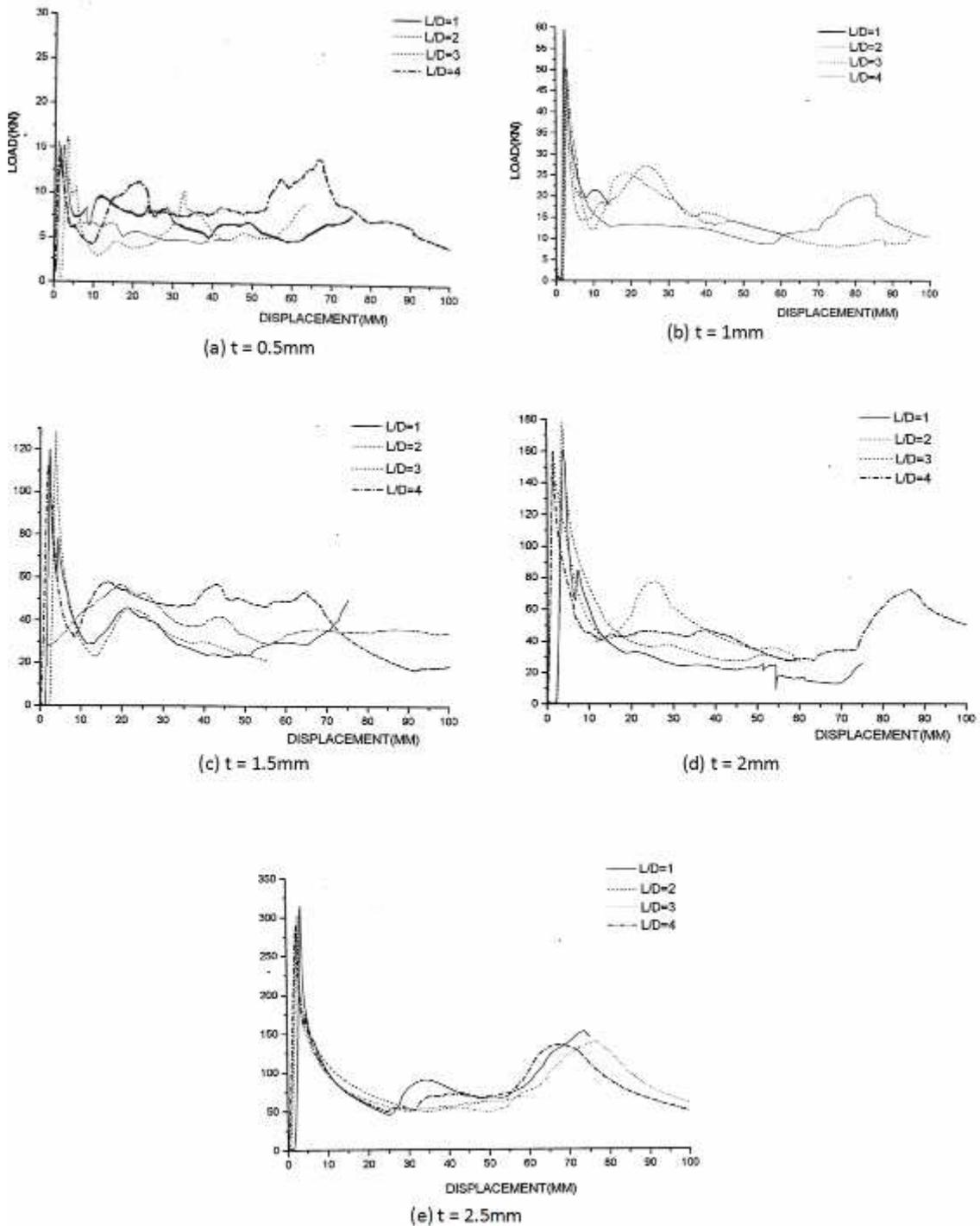
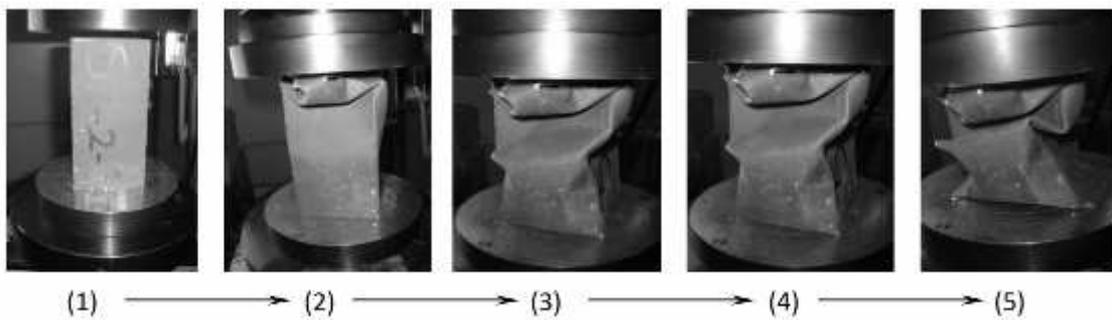


Figure 4 Effect of L/D ratio on the load displacement curves for crushed tubes with different wall thicknesses

It was observed that the curves show large initial peak load (maximum load) followed by rapid decrease in load due to the buckling of the tubes. After the initial peak, the load fluctuates and valley-to peak variations were observed according to the deformation process of the tubes. The shorter tubes show smaller valley- to peak than longer ones.

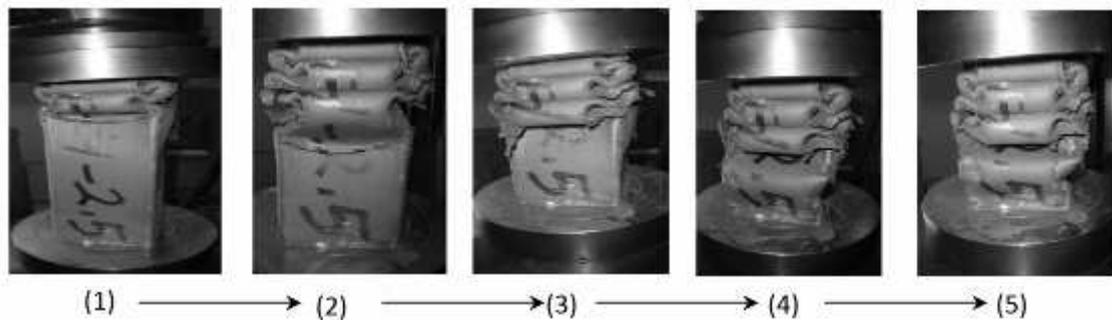
### Deformation Process

Figure (5) shows photographs of deformation process of the tube with (L/D) ratio of 2 and thickness of 1 mm. It can be seen that the tube started to buckle at the middle length due to instability of thinner of the sides and then deformed from the top end to form the first lobe of deformation, but due to resistance of the corners to crushing, it collapsed again (post buckling) at the middle of the bottom half to form the second lobe of deformation.



**Figure 5: Deformation process of the tube with (L/D) of 2 and thickness of 1mm**

Figure (6) shows the different stages of the deformation process of the tube with (L/D) of 4 and thickness of 2.5mm. It can be seen that the tube deformed with an extensional collapse mechanism described by Wierzbicki and Abramowicz [6]. The photograph (1) shows the deformation process up to the first lobe or fold, which started at the top end. Photographs (2) to (5) show the continuation of the deformation in uniform manner. Comparatively, the tube did not suffer from the instability due to the effect of thickness. This reveals that increase in the tube thickness tends to increase the instability of the tubes.



**Figure 6: Deformation process of the tube with L/D of 4 and thickness of 2.5 mm**

## Collapse Modes

Two types of collapse modes were observed experimentally during the deformation of the tubes. These are as follows:

### Regular Collapse Mode

Figure (7) shows the regular collapse mode of the tube with  $(L/D)$  of 4. As can be seen from the Figure (7a) the mode consists of three similar lobes of deformation. In each lobe, see Figure (7b), there is effectively four horizontal and eight inclined hinges at the corner edges. The fixed horizontal hinges formed around the circumference at the middle length of the tube, where two opposite sides folded inwards, while the other two opposite sides folded outwards. Figure (8) shows a sketch diagram indicates the horizontal, travel and inclined hinges for the regular mode, the inclined hinges form at AA' and traverse the shaded area at BB' at particular moment.

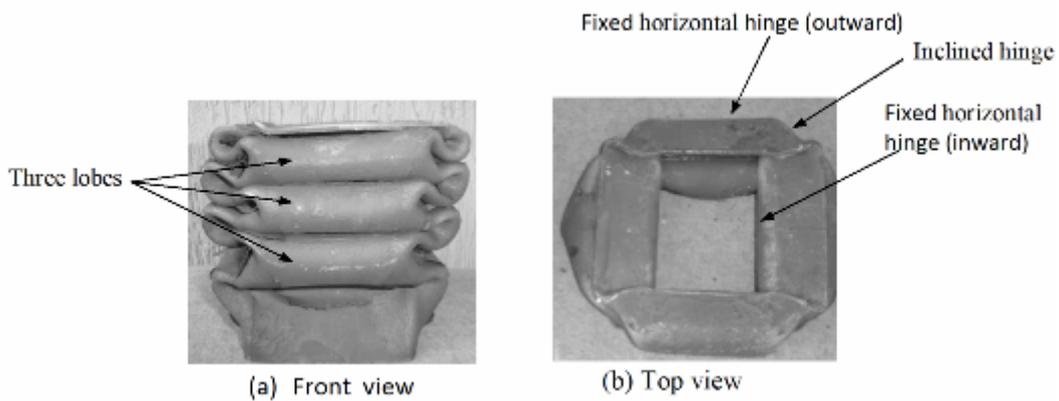


Figure 7: The regular collapse mode of the tube with  $(L/D) = 4$  and thickness = 2.5mm

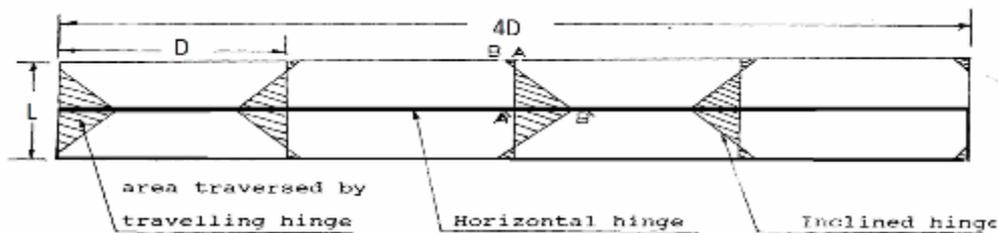


Figure 8: Development of the plastic hinges in tube wall of regular collapse mode

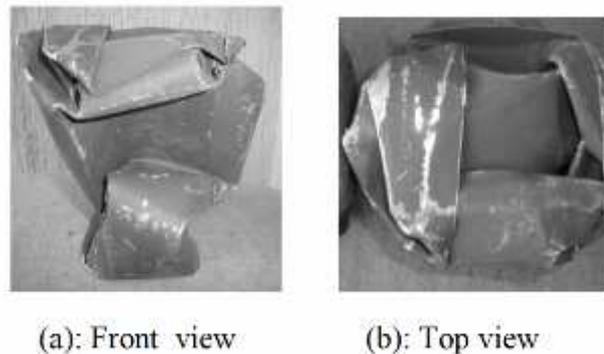


Figure 9: Irregular collapse mode of  $(L/D)$  ratio = 3 and thickness = 1.5 mm

### Effect of Wall Thickness on the Collapse Process

Figure (10) shows the effect of thickness on the behavior of the load displacement curves for tubes with different thicknesses. It can be seen that increase in the maximum load and energy absorption has been seen in all cases with increase in wall thicknesses. For example, the maximum load increases about 90% to 100% when the wall thickness increases from 2 mm to 2.5 mm. The effect of the thickness on the collapse modes is observed in all cases except for  $(L/D) < 1$ . The latter did not exhibit much variation in collapse modes. Figure (11) shows the collapse mode of crushed tubes. Clearly it can be seen that the tubes with higher thickness exhibited regular mode whereas the lower thickness exhibited irregular mode. This is due the fact that thicker tubes become more stable than the thinner ones. Furthermore, increase of energy absorption of the tubes with increase of  $L/D$  ratio and thickness can be seen in Figure (12), except one specimen with  $(L/D)$  ratio of 2 and thickness of 2.5 mm, exhibited lower energy then the other specimen with  $L/D$  ratio of 1, which probably due to the extensive buckling that lead to the tube becoming unstable during crushing.

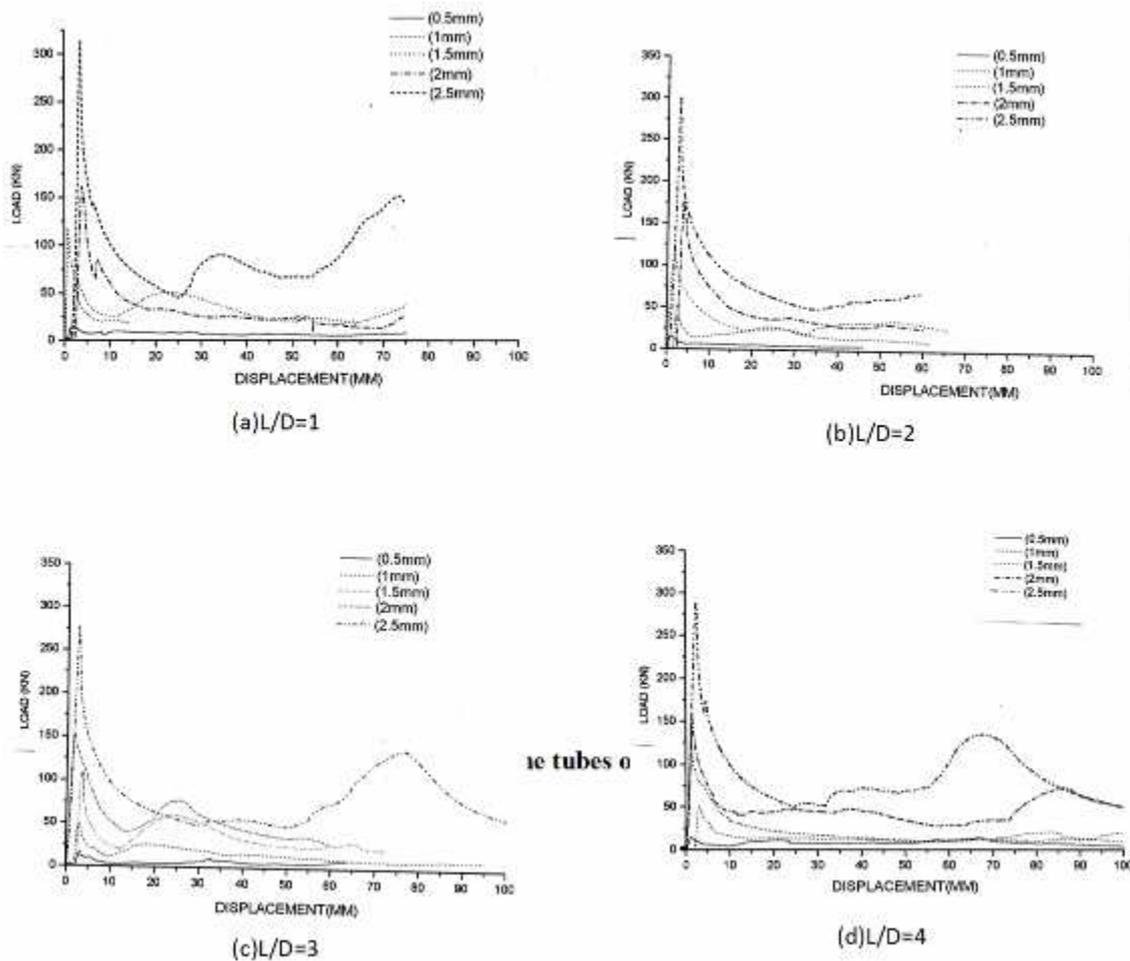


Figure 10: Effect of wall thickness on the load- displacement curves for crushed tubes with different  $L/D$  ratios with

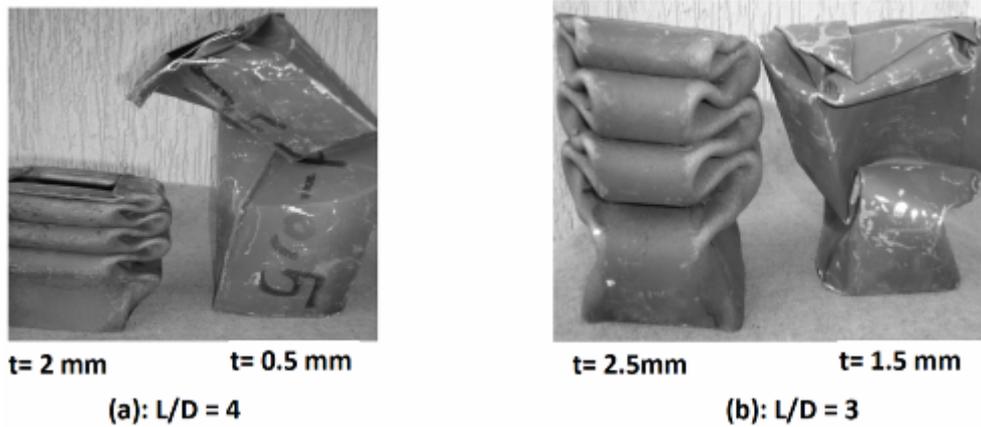


Figure 11: Effect of tube thickness on the collapse modes

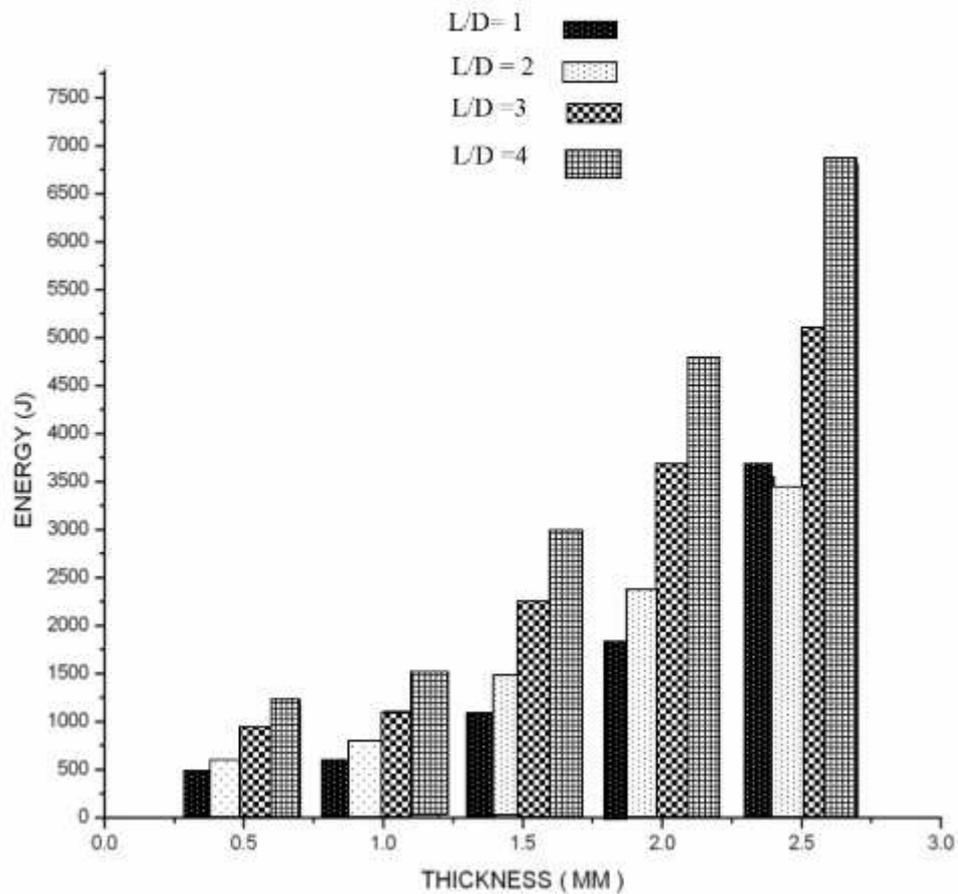


Figure 12: Thickness versus energy absorption of tubes with different L/D ratios

Figure (13) illustrates the (L/D) ratios versus mean load carrying capacity, it can be noted that the mean load carrying capacity is almost constant for thicknesses of 0.5, 1.5 and 2.5mm whereas it slightly increases for thicknesses of 1 and 2mm and (L/D) > 3. This increase is probably related to irregularity of the deformation and effect of the tube corners during the crushing.

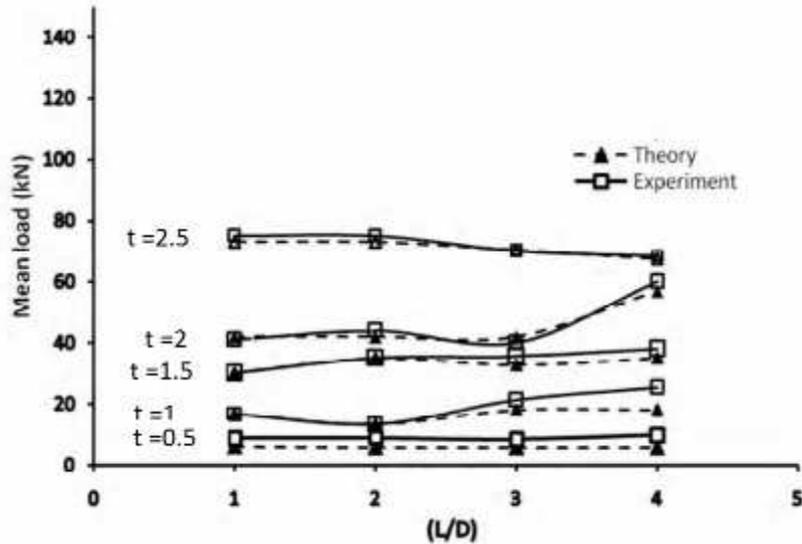


Figure 13: Mean load carrying capacity versus (L/D) ratios for tubes with different thicknesses

Figure (14) illustrates the effect of thickness of the tube on the specific energy. It can be noted that there is considerable increase of specific energy with increase of tube thickness. However, it decreases with increase of the (L/D) ratio due to the effect of the instability. This result indicated that the weight is an effective factor on the specific energy. Therefore, reducing the weight leads to get more a specific energy, reliability and stability to the tubular structures.

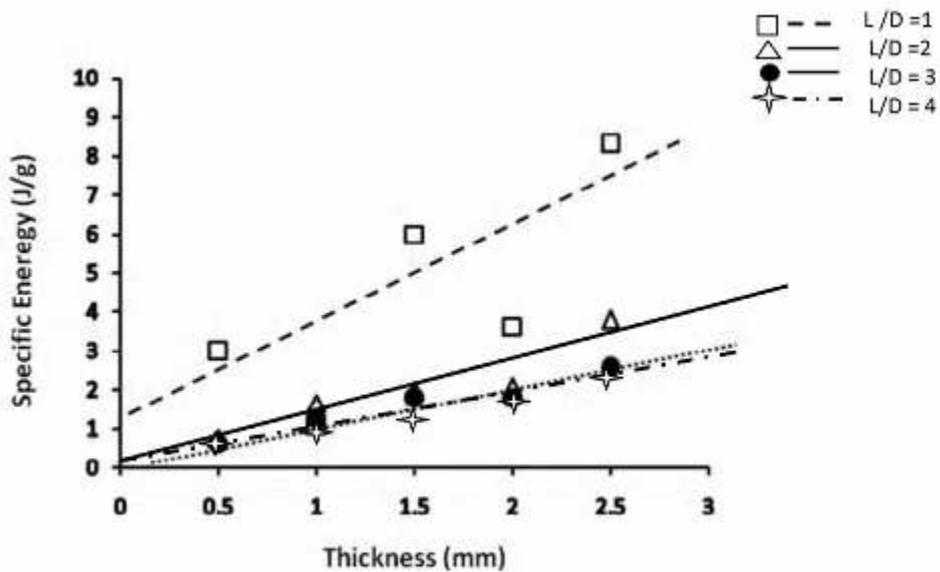


Figure 14: Specific energy versus thickness for the crushing tubes with different (L/D) ratios

### Comparison with previous work

Some results of crushed tubes in the present work were compared with the previous work in reference [4]. Taking onto account the same thickness and L/D ratio, it was found that the tubes of square cross- section exhibited lower maximum load, mean

load and energy absorption than circular cross-section tubes. This is probably related to the effect of wall corners of tubes which allowed to post-buckling to occur and therefore increase the instability of the tubes during the deformation process.

## CONCLUSION

On the basis of the results obtained, it can be concluded that the energy was absorbed by fixed, inclined and travelled plastic hinges. The tube thickness has significant effect on the collapse behavior of the tubes. The maximum load, energy absorption, and mean load carrying capacity, specific energy increased with increase of wall thickness. Two collapse modes were observed, namely regular and irregular collapsed modes. The regularity of the collapse modes increases with increase of wall thickness. Instability and post-buckling problems of the crushing tubes can be improved by using optimum tube thickness.

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### ACKNOWLEDGMENT

The authors wish to express their thanks to Engineers Mohammed Amar and Asaad Al Tomi for their help to carry out the experiment part of this work. The authors are grateful to the Research Center in at Al Falah for providing testing facilities. Appreciation and many thanks are extended to the staff members in the Research and Technical Studies Center at Abosalem – Tripoli, for their helps.

### NOMENCLATURE

$A_1, A_2$ and $A_3$	Parameters in equation (2)
$b$	Instantaneous values of half fold
$c$	Width of the element
$D$	Width of a square tube
$H$	Length of the element
$E_{ext}$	External work done
$\dot{E}_1, \dot{E}_2$ and $\dot{E}_3$	Three rates of energy dissipation
$E_{total}$	Total energy
$E_{specific}$	Specific energy
$L$	Tube length
$F$	Applied Force
$F_A$	Compressive force
$M_P$	Plastic bending moment
$M_o$	Plastic bending moment in equation (2)
$t$	Tube thickness
$P_{max}$	Maximum buckling load
$P_m$	Mean load carrying capacity
$\theta$	Subtended angle
$\bar{\sigma}$	Mean collapse stress
$\sigma_o$	Effective yield strength
SE	Square empty tube